Day 212 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 5 kg is pushed at constant speed along rough horizontal ground, $\mu = 0.25$, by a force of P N at angle θ below the horizontal, $\tan\theta = \frac{4}{3}$. Find P and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

Constant speed, so the resultant force is zero. $\sin\theta = 0.8$, $\cos\theta = 0.6$
Vertical: $R = 5g + P\sin\theta$
Horizontal: $P\cos\theta = \mu R = 0.25(5g + P\sin\theta)$
$P(0.6 - 0.2) = 0.25 \times 5g$
$P = 30.6$, $R = 73.5$
Answer: P = 30.6 N, R = 73.5 N

2 Inverse of a rational function (All boards)

$f(x) = \frac{x-4}{4x-1}$, $x \in \mathbb{R}$, $x \neq \frac{1}{4}$.

Find $f^{-1}(x)$ and state its domain.

$y = \frac{x-4}{4x-1}$: $y(4x-1) = x - 4$
$4xy - y = x - 4$
$x(4y - 1) = y - 4$
$x = \frac{y-4}{4y-1}$, then swap x and y
Answer: $f^{-1}(x) = \frac{x-4}{4x-1}$, $x \neq \frac{1}{4}$

3 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate $\int_0^2 5(xe^x)\,dx$, giving your answer to 3 s.f.

State, with a reason, whether your estimate is an overestimate or an underestimate.

$h = \frac{1}{2}$; y-values at x = 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2: 0.0000, 4.1218, 13.5914, 33.6127, 73.8906

$T = \frac{1}{4}[0.0000 + 73.8906 + 2(4.1218 + 13.5914 + 33.6127)]$

$T = 44.1$
The curve is convex ($y'' > 0$) on this interval, so the chords lie above the curve: overestimate
Answer: 44.1, an overestimate

4 Sigma notation (All boards)

Given that $\sum_{r=1}^{n}(3r + 4) = 646$, find the value of n.

$\sum_{r=1}^{n}(3r + 4) = 3 \times \frac{n(n+1)}{2} + 4n$
$\frac{3}{2}n^2 + \frac{11}{2}n = 646$
$3n^2 + 11n - 1292 = 0$
Solve and reject the negative root: n = 19
Answer: 19

5 Passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected vertically upwards from the ground. It passes a ledge 3 s after projection and again 4 s after projection. Find (a) the speed of projection, (b) the height of the ledge, (c) the greatest height. Take g = 9.8.

Same height at t = 3 and t = 4, so the stone is at its highest point at t = 3.5
(a) At the top v = 0: $0 = u - 9.8 \times 3.5$, so $u = 34.3$
(b) $s = ut - 4.9t^2$ with t = 3: $s = 34.3 \times 3 - 4.9 \times 3^2 = 58.8$
(c) $v^2 = u^2 - 2gs$: $0 = 34.3^2 - 19.6s$, $s = 60.025$
Answer: (a) 34.3 m s^{-1} (b) 58.8 m (c) 60.025 m

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6 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$\sin 3\theta + 2\cos 3\theta = 0.5$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin 3\theta \approx 3\theta$, $\cos 3\theta \approx 1 - \frac{9}{2}\theta^2$

$1 \times 3\theta + 2(1 - \frac{9}{2}\theta^2) = 0.5$

Multiply by 2 and rearrange: $18\theta^2 - 6\theta - 3 = 0$

Divide through by 3: $6\theta^2 - 2\theta - 1 = 0$

$\theta = \frac{2 \pm \sqrt{28}}{12}$, so $\theta = 0.608$

Answer: a = 6, b = −2, c = −1; $\theta \approx 0.608$

7 Quadratic from its graph (All boards)

The diagram shows the curve with equation y = f(x), where f is a quadratic.
The turning point is marked, and the curve crosses the y-axis as shown.
Find f(x), giving it in the form $ax^2 + bx + c$.

Turning point (−1, −4): $f(x) = a(x + 1)^2 - 4$

$f(0) = -6$: $a(-1)^2 - 4 = -6$

$a = -2$, then expand

Answer: $f(x) = -2x^2 - 4x - 6$

8 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The length of a plank is $N(52, 3^2)$. A plank is classed as 'special' if its length is more than 49.
9 planks are chosen at random. Find the probability that exactly 2 are special (3 s.f.).

P(special) = P(X > 49) = P(Z > −1.00) = 0.841

Y ~ B(9, 0.841)

$P(Y = 2) = C(9, 2)(0.841)^2(0.159)^7 = 0.0000645$

Answer: 0.0000645

9 Partial fractions and integration (All boards)

(a) Express $\frac{3x + 20}{(x + 4)(x + 6)}$ in partial fractions.

(b) Hence find $\int_0^4 \frac{3x + 20}{(x + 4)(x + 6)}\,dx$, giving the answer in the form ln k.

$3x + 20 \equiv A(x + 6) + B(x + 4)$: $x = -6$ gives $A = 4$; $x = -4$ gives $B = -1$

$I = [4\ln(x + 4) - \ln(x + 6)]_0^4$

$= 4\ln\frac{8}{4} - \ln\frac{10}{6}$

$= \ln\frac{48}{5}$

Answer: (a) $\frac{4}{x + 4} - \frac{1}{x + 6}$; (b) $\ln\frac{48}{5}$

10 Two-tailed binomial test critical region (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(33, p)$. H_0: $p = 0.4$ is tested against H_1: $p \neq 0.4$ at the 5% level.
(a) Find the critical region.
(b) Find the actual significance level of the test (3 s.f.).

(a) Each tail ≤ 0.025. $P(X \leq 7) = 0.0186$, $P(X \leq 8) = 0.0444$
$P(X \geq 20) = 0.0134$, $P(X \geq 19) = 0.0310$
Critical region: $X \leq 7$ or $X \geq 20$
(b) Actual level = 0.0186 + 0.0134 = 0.0319

Answer: (a) $X \leq 7$ or $X \geq 20$ (b) 0.0319