Day 211 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Binomial probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a biased coin lands heads is 0.4. In 19 independent trials, find the probability that this happens (a) exactly 8 times (b) fewer than 8 times (c) at least 8 times.

Let X ~ B(19, 0.4)
(a) P(X = 8) = 0.180
(b) P(X ≤ 7) = 0.488
(c) 1 − P(X ≤ 7) = 0.512
Answer: (a) 0.180 (b) 0.488 (c) 0.512

2 Polynomial division (All boards)

Find the quotient and the remainder when $3x^3 - 11x^2 - 20x + 4$ is divided by $(3x + 4)$.

Quotient (matching leading terms in turn): $x^2 - 5x$
$(3x + 4)(x^2 - 5x) = 3x^3 - 11x^2 - 20x$
Remainder = $3x^3 - 11x^2 - 20x + 4 - (3x^3 - 11x^2 - 20x) = 4$
Answer: quotient $x^2 - 5x$, remainder 4

3 Exponential equations reducing to quadratics (All boards)

Solve $e^{2x} - 15e^x + 54 = 0$,
giving exact answers.

Let $u = e^x$: $u^2 - 15u + 54 = 0$, so $(u - 9)(u - 6) = 0$
$e^x = 9$ or $e^x = 6$
Answer: $x = \ln 6$ or $x = 2 \ln 3$

4 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (t^3 + 6t)\mathbf{i} + (3t^2 - 12t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when t = 3, (b) the value of t when P is moving parallel to $\mathbf{i}$. Give answers to 3 s.f. if not exact.

$\mathbf{v} = d\mathbf{r}/dt = (3t^2 + 6)\mathbf{i} + (6t - 12)\mathbf{j}$
(a) t = 3: $\mathbf{v} = 33\mathbf{i} + 6\mathbf{j}$
Speed = $\sqrt{(33^2 + 6^2)} = \sqrt{1125}$
(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $6t - 12 = 0$, so t = 2
Answer: (a) 33.5 m s^{-1} (b) t = 2

5 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(27, 0.1).
Find the largest value of c such that P(X ≤ c) < 0.1.

P(X ≤ 0) = 0.0581
P(X ≤ 1) = 0.233
0.0581 < 0.1 but 0.233 > 0.1
Answer: c = 0

6 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against ln x for $y = A x^n$.
Find the value of n and the exact value of A.

ln y = ln A + n ln x: gradient n, intercept ln A
$n = \frac{-1 - 0}{2 - 1} = -1$
At (1, 0): ln A = 0 − (−1) × 1 = 1, so $A = e^1$
Answer: $n = -1$, $A = e^1$

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7 Work-energy on a rough slope (CIE)

A 4 kg block is released from rest on a rough plane inclined at α, $\tan\alpha = \frac{4}{3}$, $\mu = 0.5$, and slides 10 m down a line of greatest slope. Use work and energy to find (a) the work done against friction, (b) its speed after 10 m. Take $g = 10$.

$R = 4g\cos\alpha = 24$ N, $F = \mu R = 12$ N
(a) Work done against friction $= F \times d = 12 \times 10$
(b) Loss of PE $= 4g \times 10\sin\alpha = 320$ J
Work-energy: $\frac{1}{2}\times 4 \times v^2 = 320 - 120 = 200$
$v^2 = 100$

Answer: (a) 120 J (b) 10 m s^{-1}

8 Modulus inequalities (All boards)

Solve $|x + 8| > |-4x + 8|$.

Square both sides: $(x + 8)^2 > (-4x + 8)^2$
$((x + 8) - (-4x + 8))((x + 8) + (-4x + 8)) > 0$
$(5x)(-3x + 16) > 0$

Critical values $x = 0$ and $x = \frac{16}{3}$; sketch the (positive or negative) quadratic.

Answer: $0 < x < \frac{16}{3}$

9 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1 + 2x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.
(b) Use $x = 0.005$ to estimate $\sqrt{101}$ to 4 decimal places.

$(1 + y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, $y = 2x$

(a) $1 + x - \frac{1}{2}x^2$

$x = \frac{1}{200}$: $101 = 10^2(1 + \frac{1}{100})$ so $\sqrt{101} = 10 \times (1 + \frac{1}{100})^{\frac{1}{2}}$

$\approx 10 \times (1 + \frac{1}{200} - \frac{1}{80000}) = 10.049875$

Answer: (a) $1 + x - \frac{1}{2}x^2$ (b) 10.0499

10 Double and triple angle values (All boards)

θ is acute and $\tan\theta = 2$.

Show that $\cos 2\theta = -\frac{3}{5}$ and find the exact value of $\sin 2\theta$.

$\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1 - 2^2}{1 + 2^2} = -\frac{3}{5}$

$\sin 2\theta = \frac{2\tan\theta}{1 + \tan^2\theta} = \frac{2 \times 2}{1 + 2^2} = \frac{4}{5}$

Answer: $\cos 2\theta = -\frac{3}{5}$, $\sin 2\theta = \frac{4}{5}$