Day 210 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Tangent conditions (All boards)

Find the set of values of k for which the line $y = 2x + k$ meets the curve $y = 2x^2 + x + 1$ at two distinct points.

$2x^2 + x + 1 = 2x + k$ gives $2x^2 - x + (1 - k) = 0$

Discriminant: $(-1)^2 - 4 \times 2(1 - k) = -7 + 8k$

Two distinct points: discriminant > 0

Answer: $k > \frac{7}{8}$

2 Smallest whole number solving an exponential inequality (All boards)

£10000 is invested at 8% compound interest per year.

Find the smallest whole number of years for the value to exceed £20000.

$10000 \times 1.08^n > 20000$, so $1.08^n > 2$

$n \ln 1.08 > \ln 2$, so $n > \frac{\ln 2}{\ln 1.08} = 9.01$

$n = 10$

Answer: n = 10 years

3 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 + 6x - 8y + 16 = 0$.

(a) Find the centre and radius of the circle.

(b) Determine whether P(−1, 5) lies inside or outside the circle.

Complete the square: $(x + 3)^2 + (y - 4)^2 = 9$

Centre (−3, 4), radius 3

Distance^2 from centre to P $= 2^2 + 1^2 = 5$

Compare with $r^2 = 9$

Answer: (a) centre (−3, 4), radius 3 (b) P is inside the circle

4 Integration by parts (All boards)

Use integration by parts to find the exact value of

$\int_0^1 x\, e^{-x}\, dx$.

$u = x$, $\frac{dv}{dx} = e^{-x}$: $\int xe^{-x}\, dx = [-\frac{xe^{-x}}{1}] + \int \frac{e^{-x}}{1}\, dx$

$= [-\frac{xe^{-x}}{1} - \frac{e^{-x}}{1}]$

$= 1 - 2e^{-1}$

Answer: $1 - 2e^{-1}$

5 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 22.5 m ahead. After a reaction time of 1 s she brakes with deceleration 4 m s^{-2} and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance = 1 v, braking distance = $v^2 \div (2 \times 4)$

$1\,v + v^2 \div 8 = 22.5$, so $v^2 + 8\,v - 180 = 0$

$(v - 10)(v + 18) = 0$, so $v = 10$

(b) Braking time = 10 ÷ 4 = 2.5 s; total = 1 + 2.5

Answer: (a) 10 m s^{-1} (b) 3.5 s

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6 Venn diagram with algebra (All boards)

The Venn diagram shows the numbers of 43 students in sets A and B.
(a) Find the value of x.
(b) A student is chosen at random. Find P(A | B′).

(a) Total: (x + 5) + (x + 5) + (3x + 2) + 1 = 43
5x + 13 = 43, so x = 6
(b) Regions: 11, 11, 20, 1
Not in B: 11 + 1 = 12, of which in A: 11
Answer: (a) x = 6 (b) $\frac{11}{12}$

7 Inverse of a quadratic function (All boards)

$f(x) = x^2 - 4$, $x \geq 0$.
Find $f^{-1}(x)$ and state its domain.

$f(x) = x^2 - 4$
Let $y = f(x)$: $x^2 = y + 4$
$x \geq 0$, so take the positive root: $x = \sqrt{y + 4}$
Answer: $f^{-1}(x) = \sqrt{x + 4}$, $x \geq -4$

8 Differential equations in context (All boards)

A drink at 95°C is left in a room at 15°C. Its temperature θ °C satisfies $\frac{d\theta}{dt} = -k(\theta - 15)$.
After 5 minutes the temperature is 65°C. Find k, then the time for it to reach 25°C, to 3 s.f.

$\int \frac{1}{\theta - 15}\, d\theta = \int -k\, dt$, so $\ln(\theta - 15) = -kt + C$
$t = 0$: $\theta - 15 = 80$, so $\theta - 15 = 80e^{-kt}$
$t = 5$: $50 = 80e^{-5k}$, so $k = \frac{\ln(\frac{8}{5})}{5} = 0.0940$
$10 = 80e^{-kt}$: $t = \frac{\ln(8)}{k} = 22.1$ minutes
Answer: k = 0.0940, t = 22.1 minutes

9 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(37, p)$. H_0: $p = 0.5$ is tested against H_1: $p < 0.5$ using a 5% significance level.
(a) Find the critical region. (b) State the probability of a Type I error.
(c) Find the probability of a Type II error when $p = 0.35$ (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \leq 13$
(b) Type I error = P(reject H_0 | $p = 0.5$) = 0.0494
(c) Type II error = P(not in critical region | $p = 0.35$)
$P(X \geq 14)$ when $p = 0.35$: 0.418
Answer: (a) $X \leq 13$ (b) 0.0494 (c) 0.418

10 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (10 kg, 2 m) is smoothly hinged at A to a vertical wall, with a 6 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{4}{3}$. Find (a) the tension, (b) the magnitude of the hinge force. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about A: $T \sin\theta \times 2 = 10g \times 1 + 6g \times 2$
$T \sin\theta = (5 + 6)g = 107.8$, so (a) $T = 135$
Horizontal component of hinge force: $X = T\cos\theta = 80.9$
Vertical: $Y + T\sin\theta = (10 + 6)g$, so $Y = 10g \div 2 = 49$
(b) Force $= \sqrt{(X^2 + Y^2)} = \sqrt{(80.9^2 + 49^2)}$
Answer: (a) 135 N (b) 94.5 N