Day 21 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

10 values have $\Sigma x = 550$ and $\Sigma x^2 = 31060$.
One value was recorded as 41 but should have been 48.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 550 - 41 + 48 = 557$

$\Sigma x^2 = 31060 - 41^2 + 48^2 = 31683$

Mean $= \frac{557}{10} = 55.7$

sd $= \sqrt{\frac{31683}{10} - (55.7)^2} = 8.11$

Answer: mean 55.7, sd 8.11

2 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates $(-2, 2, -3)$, $(-2, 1, 2)$ and $(-1, -2, -2)$.
Find angle ABC, in degrees to 1 d.p.

$\text{BA} = \begin{pmatrix} 0 \\ 1 \\ -5 \end{pmatrix}$, $\text{BC} = \begin{pmatrix} 1 \\ -3 \\ -4 \end{pmatrix}$

$\text{BA} \cdot \text{BC} = 17$

$|\text{BA}| = \sqrt{26}$, $|\text{BC}| = \sqrt{26}$

$\cos \text{ABC} = \frac{17}{\sqrt{26} \times \sqrt{26}} = 0.6538$, so angle ABC = 49.2°

Answer: 49.2°

3 Arithmetic series: finding n (All boards)

An arithmetic series has first term 7 and common difference 4.
The sum of the first n terms is 525. Find n.

$\frac{n}{2}(2 \times 7 + (n - 1) \times 4) = 525$

$4n^2 + 10n = 1050$

$4n^2 + 10n - 1050 = 0$

$(n - 15)(4n + 70) = 0$, so $n = 15$ or $n = -\frac{35}{2}$

n must be a positive integer, so n = 15

Answer: 15

4 Pulled at an angle on rough ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 12 kg box is pulled along rough horizontal ground, $\mu = \frac{1}{3}$, by a rope at angle θ above the horizontal, where $\tan\theta = \frac{5}{12}$.
The tension is 40 N. Find (a) the normal reaction, (b) the acceleration of the box. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

(a) Vertical: $R + 40\sin\theta = 12g$, so $R = \frac{6644}{65}$

(b) Horizontal: $40\cos\theta - \mu R = 12a$

$\frac{480}{13} - \frac{1}{3} \times \frac{6644}{65} = 12a$

$a = \frac{139}{585}$

Answer: (a) 102 N (b) 0.238 m s^{-2}

5 Symmetric interval about the mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the height (cm), is N(179, 7^2).
Find the symmetric limits about the mean between which the central 99% of values lie (3 s.f.).

Each tail has area 0.005, so P(Z < z) = 0.995
z = 2.58
Limits: 179 ± 2.58 × 7
= 161 and 197
Answer: 161 to 197

6 Velocity, rest and total distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on the x-axis with velocity v = ($6t^2 - 18t + 12$) m s^{-1} at time t s, t ≥ 0, and starts at O. Find (a) the times when P is at instantaneous rest, (b) the total distance P travels in the first 4 s.

(a) v = 6(t − 1)(t − 2) = 0, so t = 1 or t = 2
(b) Displacement x = ∫v dt = $2t^3 - 9t^2 + 12t$
x(1) = 5, x(2) = 4, x(4) = 32
Direction changes at t = 1 and t = 2, so distance = |5| + |4 − 5| + |32 − 4|
Answer: (a) t = 1, t = 2 (b) 34 m

7 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (3, −6). Its centre lies in the fourth quadrant.
Find the equations of the two possible circles.

Centre (r, −r), radius r > 0.
Through (3, −6): $(3 - r)^2 + (-6 + r)^2 = r^2$
$r^2 - 18r + 45 = 0$
(r − 3)(r − 15) = 0, so r = 3 or 15
Answer: $(x - 3)^2 + (y + 3)^2 = 9$ or $(x - 15)^2 + (y + 15)^2 = 225$

8 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against x for $y = A e^{kx}$.
Find the exact values of A and k.

ln y = ln A + kx, a straight line with gradient k and intercept ln A
Intercept ln A = 1, so $A = e^1$
Gradient $k = \frac{3 - 1}{4 - 0} = \frac{1}{2}$
Answer: $A = e^1$, $k = \frac{1}{2}$

9 Integration by parts (All boards)

Use integration by parts twice to find the exact value of

$\int_0^2 x^2 e^x \, dx$.

$\int x^2 e^x \, dx = x^2 e^x - \int 2x e^x \, dx$
$\int x e^x \, dx = x e^x - e^x$, so $\int x^2 e^x \, dx = e^x(x^2 - 2x + 2)$
$= 2e^2 - 2$
Answer: $2e^2 - 2$

10 Circle touching an axis (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows a circle that touches the x-axis at A(−3, 0) and passes through B(1, 4).
Find the equation of the circle.

Touches the x-axis at A, so the centre is (−3, r) and the radius is r.
B on the circle: $(4)^2 + (4 - r)^2 = r^2$
16 + 16 − 8r = 0 so r = 4
Answer: $(x + 3)^2 + (y - 4)^2 = 16$