Day 209 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(−1, −7) and B(7, −1) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(0, −8) lies on the circle.

Centre = midpoint of AB = (3, −4)
$r^2 = (-4)^2 + (-3)^2 = 25$
$(x - 3)^2 + (y + 4)^2 = 25$: $x^2 + y^2 - 6x + 8y = 0$
(b) $CM^2 = (-3)^2 + (-4)^2 = 25 = r^2$
Answer: (a) $x^2 + y^2 - 6x + 8y = 0$

2 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 3 m s^{-1} and accelerates uniformly. She travels 90 m in the next 10 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $90 = 3\times10 + \frac{1}{2}\times a\times10^2$, so $a = 1.2$
(b) $v = u + at = 3 + 1.2\times10 = 15$
(c) For the midpoint, $s = 45$: $v^2 = u^2 + 2as = 3^2 + 2\times1.2\times45$
$v^2 = 117$
Answer: (a) 1.2 m s^{-2} (b) 15 m s^{-1} (c) 10.8 m s^{-1}

3 Discriminant: range of k (All boards)

The equation $x^2 + (2k - 4)x + (k - 2) = 0$ has two distinct real roots.
Find the set of possible values of k.

Discriminant: $b^2 - 4ac = (2k - 4)^2 - 4(k - 2)$
$= 4k^2 - 20k + 24$
Roots of the quadratic in k: k = 2 and k = 3
Required: $b^2 - 4ac > 0$
Answer: $k < 2$ or $k > 3$

4 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $3x^2 + 2xy - 2x - 3y = 15$.
Show that P(−1, −2) lies on the curve and find the equation of the tangent at P in the form ax + by + c = 0 with integer a, b, c.

Differentiate (product rule on the xy term): $(6x + 2y - 2) + (2x - 3)\frac{dy}{dx} = 0$

At P(−1, −2): $-12 + (-5)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{12}{5}$

Tangent: $y + 2 = (-\frac{12}{5})(x + 1)$, i.e. $12x + 5y + 22 = 0$

Answer: $12x + 5y + 22 = 0$

5 Interpreting a regression line (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows data and the regression line y = 30 + 1.2x.
(a) Interpret the gradient.
(b) Estimate y when x = 11 and comment.

(a) On average, y increases by 1.2 m for each extra 1 m/s of x
(b) $y = 30 + 1.2 \times 11 = 43.2$
x = 11 is within the range of the data (5 to 14): interpolation, so reliable
Answer: (a) y increases by 1.2 m per m/s (b) 43.2, reliable

6 Integration by parts (All boards)

Use integration by parts twice to find the exact value of

$\int_0^1 x^2e^x\,dx$.

$\int x^2e^x\,dx = x^2e^x - \int 2xe^x\,dx$

$\int xe^x\,dx = xe^x - e^x$, so $\int x^2e^x\,dx = e^x(x^2 - 2x + 2)$

$= e - 2$

Answer: $e - 2$

7 Hypothesis test for a Poisson mean (CIE)

Faults occur as a Poisson process at a mean rate of 4 per roll of cloth.
After a change, the number recorded in one roll of cloth is 5.
Test at the 1% level whether the mean rate has increased.

$H_0: \lambda = 4$, $H_1: \lambda > 4$; $X \sim Po(4)$

$P(X \geq 5) = 1 - P(X \leq 4) = 0.371$

$0.371 > 0.01$

Do not reject H_0: insufficient evidence of a change.

Answer: p-value 0.371; do not reject H_0

8 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

f(x) = 2 arcsin(2x).

State the domain and range of f, and find $f^{-1}(x)$.

arcsin has domain -1 to 1, so $-1 \leq 2x \leq 1$

Range of arcsin is $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, so f(x) runs from $-\frac{2\pi}{2}$ to $\frac{2\pi}{2}$

$y = 2\arcsin(2x)$ gives $\sin\frac{y}{2} = 2x$, so $f^{-1}(x) = \frac{1}{2}\sin\frac{x}{2}$

Answer: domain $-\frac{1}{2} \leq x \leq \frac{1}{2}$, range $-\frac{2\pi}{2} \leq f(x) \leq \frac{2\pi}{2}$, $f^{-1}(x) = \frac{1}{2}\sin\frac{x}{2}$

9 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1 + 2x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.

(b) Use $x = 0.02$ to estimate $\sqrt{26}$ to 3 decimal places.

$(1 + y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, $y = 2x$

(a) $1 + x - \frac{1}{2}x^2$

$x = \frac{1}{50}$: $26 = 5^2(1 + \frac{1}{25})$ so $\sqrt{26} = 5 \times (1 + \frac{1}{25})^{\frac{1}{2}}$

$\approx 5 \times (1 + \frac{1}{50} - \frac{1}{5000}) = 5.099000$

Answer: (a) $1 + x - \frac{1}{2}x^2$ (b) 5.099

10 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (3 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (7 kg), hanging 0.5 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: $7g - T = 7a$

A: $T = 3a$

Adding: $7g = 10a$, so $a = 6.86$

(a) $T = 3 \times 6.86 = 20.58$

(b) $v^2 = 0 + 2 \times 6.86 \times 0.5 = 6.86$

Answer: (a) 20.58 N (b) 2.62 m s^{-1}