Day 208 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (5, 2, −1), (4, −2, −2) and (−2, 0, 1).
Find angle ABC, in degrees to 1 d.p.

$\text{BA} = \begin{pmatrix}1\\4\\1\end{pmatrix}$, $\text{BC} = \begin{pmatrix}-6\\2\\3\end{pmatrix}$

$\text{BA}\cdot\text{BC} = 5$

$|\text{BA}| = \sqrt{18}$, $|\text{BC}| = \sqrt{49}$

$\cos \text{ABC} = \frac{5}{\sqrt{18} \times \sqrt{49}} = 0.1684$, so angle ABC = 80.3°

Answer: 80.3°

2 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (6 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (8 kg), hanging 1 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: $8g - T = 8a$

A: $T = 6a$

Adding: $8g = 14a$, so $a = 5.6$

(a) $T = 6 \times 5.6 = 33.6$

(b) $v^2 = 0 + 2 \times 5.6 \times 1 = 11.2$

Answer: (a) 33.6 N (b) 3.35 m s^{-1}

3 Differentiating a^x (All boards)

$y = 2 \times 3^x$.
Find the exact gradient of the curve at x = 1 and the equation of the tangent there.

$\frac{dy}{dx} = 2 \times 1 \ln 3 \times 3^x = 2 \ln 3 \times 3^x$

At x = 1: y = 6 and gradient = $6 \ln 3$

Tangent: $y = (6 \ln 3)x + 6 - 6 \ln 3$

Answer: gradient $6 \ln 3$; $y = (6 \ln 3)x + 6 - 6 \ln 3$

4 Venn diagrams and conditional probability (All boards)

P(A) = 0.65, P(B) = 0.3 and P(A ∩ B) = 0.15.
Find
(a) P(A ∪ B)
(b) P(A | B)
(c) P(B | A)

(a) $\text{P}(\text{A} \cup \text{B}) = \text{P}(\text{A}) + \text{P}(\text{B}) - \text{P}(\text{A} \cap \text{B}) = 0.65 + 0.3 - 0.15 = 0.8$

(b) $\text{P}(\text{A} \mid \text{B}) = \frac{\text{P}(\text{A} \cap \text{B})}{\text{P}(\text{B})} = \frac{0.15}{0.3} = 0.5$

(c) $\text{P}(\text{B} \mid \text{A}) = \frac{\text{P}(\text{A} \cap \text{B})}{\text{P}(\text{A})} = \frac{0.15}{0.65} = \frac{3}{13}$

Answer: (a) 0.8 (b) 0.5 (c) $\frac{3}{13}$

5 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2 x + 3\sin x - 3 = 0$
for $0° \le x \le 180°$.

Use $\cos^2 x \equiv 1 - \sin^2 x$: $2\sin^2 x - 3\sin x + 1 = 0$

Solving the quadratic: $\sin x = \frac{1}{2}$ or $\sin x = 1$

$\sin x = \frac{1}{2}$: $x = 30°, 150°$

$\sin x = 1$: $x = 90°$

Answer: x = 30°, 90°, 150°

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6 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{7} - 3) = -5 + 2\sqrt{7}$, writing x in the form $p + q\sqrt{7}$.

$x = \dfrac{-5 + 2\sqrt{7}}{\sqrt{7} - 3}$

Multiply top and bottom by $\sqrt{7} + 3$.

Numerator: $(-5 + 2\sqrt{7})(\sqrt{7} + 3) = -1 + \sqrt{7}$

Denominator: $7 - 9 = -2$

Answer: $x = \frac{1}{2} - \frac{1}{2}\sqrt{7}$

7 Smooth incline held by a string (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle rests on a smooth plane inclined at α, $\tan\alpha = \frac{3}{4}$, held by a string in the vertical plane through a line of greatest slope, at angle θ above the slope, $\tan\theta = \frac{5}{12}$. Find the tension and the normal reaction. Take g = 9.8.

$\sin\alpha = 0.6$, $\cos\alpha = 0.8$, $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

Along the plane: $T\cos\theta = 3g\sin\alpha$, so $T = 19.1$

Perpendicular: $R + T\sin\theta = 3g\cos\alpha$

$R = 3g\cos\alpha - T\sin\theta = 23.5 - 19.1\times0.385$

Answer: T = 19.11 N, R = 16.17 N

8 Tangent conditions (All boards)

The line $y = kx - 3$ is a tangent to the curve $y = x^2 - x - 2$.

(a) Find the possible values of k.

(b) For the larger value of k, find the coordinates of the point of contact.

Equate: $x^2 - x - 2 = kx - 3$

$x^2 + (-1 - k)x + 1 = 0$

Tangent: discriminant = 0: $(-1 - k)^2 = 4$

$-1 - k = \pm2$ so $k = 1$ or -3

$k = 1$: $x = \dfrac{k - (-1)}{2} = 1$, $y = -2$

Answer: (a) k = −3 or k = 1 (b) (1, −2)

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $x^2 - y^2 = 26$.

Assume $x^2 - y^2 = (x - y)(x + y) = 26$.

$x - y$ and $x + y$ are both odd or both even (they differ by 2y).

Both odd: the product is odd, but 26 is even.

Both even: the product is a multiple of 4, but 26 is not.

Contradiction either way.

Answer: no such integers exist

10 Normal approximation to the Poisson (CIE)

X ~ Po(20). Use a suitable approximation to find $P(X \geq 15)$, to 3 significant figures.

$\lambda > 15$, so Y ~ N(20, 20) approximately

Continuity correction: P(Y > 14.5)

$z = -1.23$, so probability = 0.891

Answer: 0.891