Day 207 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Perpendicular bisector and area (All boards)

A is (−2, 2) and B is (4, −2).
(a) Find the equation of the perpendicular bisector of AB, in the form ax + by = c.
(b) The perpendicular bisector meets the x-axis at P. Find the area of triangle ABP.

Midpoint of AB = (1, 0); gradient AB = $-\frac{2}{3}$

Perpendicular gradient = $\frac{3}{2}$

Equation: $3x - 2y = 3$
$y = 0$: P = (1, 0)
Area = ½ |(4 − (−2))(0 − 2) − (1 − (−2))(−2 − 2)|
Answer: (a) $3x - 2y = 3$ (b) 0

2 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 26 m ahead. After a reaction time of 0.6 s she brakes with deceleration 2.5 m s^{-2} and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance = 0.6 v, braking distance = $v^2 \div (2\times2.5)$
$0.6v + v^2 \div 5 = 26$, so $v^2 + 3v - 130 = 0$
$(v - 10)(v + 13) = 0$, so $v = 10$
(b) Braking time = 10 ÷ 2.5 = 4 s; total = 0.6 + 4
Answer: (a) 10 m s^{-1} (b) 4.6 s

3 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations to find the value that
$\frac{1 - \cos 3\theta}{\theta \sin 4\theta}$
approaches as $\theta \to 0$.

$1 - \cos 3\theta \approx \frac{1}{2}(3\theta)^2 = \frac{9}{2}\theta^2$

$\sin 4\theta \approx 4\theta$

Expression $\approx \frac{\frac{9}{2}\theta^2}{\theta \times 4\theta} = \frac{9}{8} = \frac{9}{8}$

Answer: $\frac{9}{8}$

4 Selections with combinations (OCR, MEI, Eduqas, CIE)

A group of 6 boys and 8 girls. A team of 3 is chosen at random.
Find the probability that the team has exactly 2 of the girls.

Total teams = C(14, 3) = 364
Favourable = C(8, 2) × C(6, 1) = 28 × 6 = 168
$P = \frac{168}{364}$
$= \frac{6}{13}$
Answer: $\frac{6}{13}$

5 R-form and solving a sin x + b cos x = c (All boards)

(a) Express $12\sin\theta - 16\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give α to 1 d.p.
(b) Solve $12\sin\theta - 16\cos\theta = -18$ for $0° \le \theta < 360°$, giving answers to 1 d.p.

$R^2 = 12^2 + 16^2 = 400$, so R = 20; $\tan\alpha = \frac{16}{12}$ gives α = 53.1°

$20\sin(\theta - 53.1°) = -18$, so $\sin(\theta - 53.1°) = -\frac{9}{10}$

Principal value −64.2°, then use the symmetry of the sine graph
θ = 297.3°, 349.0°
Answer: (a) R = 20, α = 53.1° (b) θ = 297.3°, 349.0°

6 Normal approximation to the binomial (All boards)

X ~ B(120, 0.2). Use a normal approximation to estimate P(27 ≤ X ≤ 35) to 3 significant figures.

np = 24, npq = 19.2, so Y ~ N(24, 19.2)
With continuity correction: P(26.5 < Y < 35.5)
z-values 0.571 and 2.62, so probability = 0.280
Answer: 0.280

7 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^7 in the expansion of

$(3x^3 - \frac{3}{x})^5$.

General term: ${}^5C_r\,(3x^3)^{5-r}\,(-3x^{-1})^r$
Power of x: $3(5 - r) - 1r = 7$
$15 - 4r = 7$ so $r = 2$
Coefficient = ${}^5C_2 \times 3^3 \times (-3)^2 = 2430$
Answer: 2430

8 Greatest and least values using R-form (All boards)

$g(\theta) = \frac{1}{9 + 4\sin\theta + 3\cos\theta}$

(a) Write $4\sin\theta + 3\cos\theta$ in the form $R\sin(\theta + \alpha)$, with α acute.
(b) Find the greatest and least values of $g(\theta)$ and the smallest positive θ at which the greatest value occurs (1 d.p.).

$R = \sqrt{4^2 + 3^2} = 5$, $\alpha = \tan^{-1}\frac{3}{4} = 36.9°$

Denominator = $9 + 5\sin(\theta + 36.9°)$, ranging from 4 to 14

Greatest g = $\frac{1}{4}$ (denominator least), least g = $\frac{1}{14}$

Least denominator when $\sin(\theta + 36.9°) = -1$: $\theta + 36.9° = 270°$
$\theta = 233.1°$

Answer: R = 5, α = 36.9°; greatest $\frac{1}{4}$, least $\frac{1}{14}$; θ = 233.1°

9 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (−8, 9). Its centre lies in the second quadrant. Find the equations of the two possible circles.

Centre (−r, r), radius r > 0.
Through (−8, 9): $(-8 + r)^2 + (9 - r)^2 = r^2$
$r^2 - 34r + 145 = 0$
$(r - 5)(r - 29) = 0$, so r = 5 or 29
Answer: $(x + 5)^2 + (y - 5)^2 = 25$ or $(x + 29)^2 + (y - 29)^2 = 841$

10 Range of horizontal force on a rough slope (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg particle is in equilibrium on a rough plane inclined at α, $\tan\alpha = \frac{5}{12}$, $\mu = \frac{1}{3}$. A horizontal force of P N, in the vertical plane through a line of greatest slope, pushes it towards the plane. Find the least and greatest values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$, weight W = 39.2 N. Normal reaction $R = W\cos\alpha + P\sin\alpha$
Least P: about to slide down, friction $F = \mu R$ up the slope
$P\cos\alpha + \mu(W\cos\alpha + P\sin\alpha) = W\sin\alpha \rightarrow P = W(\sin\alpha - \mu\cos\alpha) \div (\cos\alpha + \mu\sin\alpha) = 2.87$
Greatest P: about to slide up, friction down the slope
$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) \rightarrow P = W(\sin\alpha + \mu\cos\alpha) \div (\cos\alpha - \mu\sin\alpha) = 34.1$
Answer: least 2.87 N, greatest 34.1 N