Day 206 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

A circular ripple has area A cm^2 increasing at a constant 8 cm^2s^{-1}.
Find the rate at which the circumference is increasing when the radius is 3 cm.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$ and $\frac{dr}{dt} = \frac{8}{2\pi r}$

$C = 2\pi r$, so $\frac{dC}{dt} = 2\pi \times \frac{dr}{dt} = 2\pi \times \frac{8}{2\pi \times 3} = \frac{8}{3}$

Answer: $\frac{8}{3}$ cm/s

2 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 300 kg carries a person of mass 50 kg. The cable tension is 3535 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 350 kg), taking upwards as positive: $T - 350g = 350a$
$3535 - 3430 = 350a$, so $a = 0.3$

(a) a is positive, so the acceleration is 0.3 m s^{-2} upwards
(b) Person: $R - 50g = 50a$, so $R = 50(9.8 + 0.3)$

Answer: (a) 0.3 m s^{-2} upwards (b) 505 N

3 Chain rule and gradients (All boards)

$y = \sqrt{3x^2 - 11}$.
Find $\frac{dy}{dx}$ and the gradient of the curve at x = 2.

$y = (3x^2 - 11)^{\frac{1}{2}}$, so $\frac{dy}{dx} = \frac{1}{2}(3x^2 - 11)^{-\frac{1}{2}} \times 6x = \frac{3x}{\sqrt{3x^2 - 11}}$

At x = 2: $\sqrt{3x^2 - 11} = 1$, so gradient $= \frac{6}{1} = 6$

Answer: $\frac{dy}{dx} = \frac{3x}{\sqrt{3x^2 - 11}}$; gradient = 6

4 Conditional probability and independence (All boards)

P(B) = 0.4, P(A) = 0.3 and P(A | B) = 0.45.
(a) Find $P(A \cap B)$ and $P(A \cup B)$.
(b) Find P(B | A).
(c) Determine whether A and B are independent.

(a) $P(A \cap B) = P(A \mid B)P(B) = 0.45 \times 0.4 = 0.18$
$P(A \cup B) = 0.3 + 0.4 - 0.18 = 0.52$

(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.18}{0.3} = 0.6$

(c) $P(A)P(B) = 0.3 \times 0.4 = 0.12$, which does not equal $P(A \cap B) = 0.18$
So A and B are not independent

Answer: (a) 0.18, 0.52 (b) 0.6 (c) not independent

5 Area and perimeter of a segment (exact) (All boards)

The circle has centre O and radius 18 cm. Angle AOB = $\frac{\pi}{3}$.
Find the exact area and the exact perimeter of the shaded segment.

Sector area $= \frac{1}{2}r^2\theta = 54\pi$; triangle OAB $= \frac{1}{2}r^2\sin\theta = 81\sqrt{3}$

Segment area = sector − triangle $= 54\pi - 81\sqrt{3}$

Chord AB $= 2r\sin\frac{\theta}{2} = 18$; arc $= r\theta = 6\pi$

Perimeter = arc + chord $= 6\pi + 18$

Answer: area $= 54\pi - 81\sqrt{3}$ cm^2, perimeter $= 6\pi + 18$ cm

6 Sum to infinity: two possible ratios (All boards)

The second term of a geometric series is 6 and its sum to infinity is 25.
Find the two possible values of the first term.

$ar = 6$ and $\frac{a}{(1-r)} = 25$

$a = 25(1-r)$, so $25(1-r)\,r = 6$

$25r^2 - 25r + 6 = 0$: $r = \frac{2}{5}$ or $r = \frac{3}{5}$

$a = 25(1-r) = 15$ or 10

Answer: a = 10 or a = 15

7 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bulbs are defective with probability 0.15, independently. A pack of 8 bulbs is rejected if it has at least 3 defective.
From 8 such packs, find the probability that exactly 2 are rejected (3 s.f.).

$Y \sim B(8, 0.15)$. $P(\text{rejected}) = P(Y \geq 3) = 1 - 0.895 = 0.105$

$W \sim B(8, 0.105)$ is the number rejected

$P(W = 2) = C(8, 2)(0.105)^2(1 - 0.105)^6 = 0.159$

Answer: 0.159

8 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 12 and the coefficient of x^2 is 54.
Find n and k.

$nk = 12$ and $\frac{n(n-1)}{2}k^2 = 54$

$(n-1)k = \frac{2 \times 54}{12} = 9$ (dividing the second equation by the first)

$n\,k - (n-1)\,k = k = 12 - 9 = 3$

$n = \frac{12}{3} = 4$

Answer: n = 4, k = 3

9 Closest approach of two particles (Edexcel, OCR, MEI)

At t = 0, P and Q have position vectors $(17\mathbf{i} + 2\mathbf{j})$ m and $(2\mathbf{i} - \mathbf{j})$ m and velocities $(-2\mathbf{i} + 2\mathbf{j})$ m s^{-1} and $(\mathbf{i} - \mathbf{j})$ m s^{-1}. Find (a) the time when they are closest, (b) the least distance between them. Give answers to 3 s.f. if not exact.

$\mathrm{PQ} = r_P - r_Q = (15\mathbf{i} + 3\mathbf{j}) + t(-3\mathbf{i} + 3\mathbf{j})$

$d^2 = (-3t + 15)^2 + (3t + 3)^2 = 18t^2 - 72t + 234$

(a) $d(d^2)/dt = 0$: $36\,t + -72 = 0$, so $t = 2$

(b) At $t = 2$: $\mathrm{PQ} = 9\mathbf{i} + 9\mathbf{j}$, so $d = \sqrt{162}$

Answer: (a) t = 2 s (b) 12.7 m

10 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $3x - 4y = -21$ meets the circle $(x-3)^2 + (y-3)^2 = 25$ at the points A and B. Find the exact length of AB.

Centre (3, 3), radius 5. Distance from centre to line:

$d = \frac{|18|}{5} = \frac{18}{5}$

$\text{Half-chord}^2 = 25 - (\frac{18}{5})^2 = \frac{301}{25}$

$\mathrm{AB} = 2\sqrt{\frac{301}{25}}$

Answer: $\frac{10\sqrt{301}}{25}$