Day 205 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show by counter-example that the statement "$n^2 + n + 7$ is prime for all positive integers n" is false.

Try n = 6:

$6^2 + 6 + 7 = 49 = 7^2$

This is not prime.

Answer: n = 6 gives $49 = 7^2$, not prime

2 Double angle equations (All boards)

Solve $\cos 2x + 7\cos x = -4$
for $0° \leq x \leq 360°$.

$\cos 2x \equiv 2\cos^2 x - 1$: $2\cos^2 x + 7\cos x + 3 = 0$

Solve the quadratic: $\cos x = -\frac{1}{2}$ or $\cos x = -3$

$\cos x = -\frac{1}{2}$: $x = 120°, 240°$

$\cos x = -3$ is impossible

Answer: x = 120°, 240°

3 Particles on two smooth inclined planes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest. Find (a) the magnitude of the acceleration and which particle moves down, (b) the tension. Take g = 9.8. Give answers to 3 s.f. if not exact.

A: weight component along plane = 4 g sin 30°; B: 3 g sin 60°

B moves down, A up (3 sin 60° > 4 sin 30°): F = ma for the system

3 g sin 60° − 4 g sin 30° = (4 + 3)a

(a) a = 0.837

(b) For A: T − 4 g sin 30° = 4 a, so T = 22.9

Answer: (a) 0.837 m s^{-2}, B moves down (b) 22.9 N

4 Binomial expansion: rational powers (All boards)

Expand $(1 + 2x)^{-\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3.
State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \ldots$ with $n = -\frac{3}{2}$, $y = 2x$

Coefficients of 1, x, x^2, x^3: 1, -3, $\frac{15}{2}$, $-\frac{35}{2}$

Valid when $|2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 - 3x + \frac{15}{2}x^2 - \frac{35}{2}x^3$; $|x| < \frac{1}{2}$

5 Separable differential equations (All boards)

Solve the differential equation

$y\frac{dy}{dx} = 4x$,

given that y = 2 when x = 0 (y > 0). Give y in terms of x.

Separate: $\int y\,dy = \int 4x\,dx$, so $\frac{y^2}{2} = 2x^2 + C$

x = 0, y = 2 gives C = 2

$y^2 = 4x^2 + 4$

Answer: $y = \sqrt{4x^2 + 4}$

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6 Normal approximation to the binomial (All boards)

X ~ B(80, 0.4). Use a normal approximation to estimate P(X ≥ 36) to 3 significant figures.

np = 32, npq = 19.2, so Y ~ N(32, 19.2)
With continuity correction: P(Y > 35.5)
$z = \frac{35.5 - 32}{4.38} = 0.799$, P(Z > 0.799) = 0.212
Answer: 0.212

7 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 8 kg box is pulled from rest along rough horizontal ground by a horizontal force of 25.12 N and moves 1.6 m in 4 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = ½at^2$: $1.6 = ½a \times 4^2$, so a = 0.2
(a) P − μ8 g = 8 a: 25.12 − 78.4 μ = 1.6, so μ = 0.3
(b) v = at = 0.2×4 = 0.8; after removal deceleration = μg = 2.94
$0 = 0.8^2 - 2 \times 2.94 \times s$, s = 0.109
Answer: (a) μ = 0.3 (b) 0.109 m

8 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2\,\theta + 14\cot\theta + 2 = 0$
for $-180° < \theta < 180°$, $\theta \neq 0$. Give answers to 1 d.p.

Use $\text{cosec}^2\,\theta \equiv 1 + \cot^2\theta$: $4\cot^2\theta + 14\cot\theta + 6 = 0$
$\cot\theta = -3$ or $\cot\theta = -\frac{1}{2}$
$\cot\theta = -3$ means $\tan\theta = -\frac{1}{3}$: $\theta = -18.4°, 161.6°$
$\cot\theta = -\frac{1}{2}$ means $\tan\theta = -2$: $\theta = -63.4°, 116.6°$
Answer: θ = −63.4°, −18.4°, 116.6°, 161.6°

9 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{14x + 2}{(1 - 3x)(1 + x)}$.
(a) Express f(x) in partial fractions.
(b) Hence expand f(x) up to x^2.
(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - 3x)} + \frac{B}{(1 + x)}$: A = 5, B = −3
(b) $5(1 - 3x)^{-1} = 5(1 + 3x + 9x^2 \ldots)$
Add the series for both fractions.
(c) Need |3x| < 1 and |x| < 1
Answer: (a) $\frac{5}{(1 - 3x)} + \frac{-3}{(1 + x)}$ **(b)** $2 + 18x + 42x^2$ **(c)** $|x| < \frac{1}{3}$

10 Continuous random variable: cumulative distribution and median (CIE)

The random variable X has pdf $f(x) = \frac{3}{27}x^2$ for $0 \leq x \leq 3$, and 0 otherwise.

(a) Find the cumulative distribution function F(x) for $0 \leq x \leq 3$.

(b) Find $P(X > \frac{3}{2})$. (c) Find the median of X to 3 s.f.

(a) $F(x) = \int_0^x f(t)\,dt = \frac{x^3}{27}$

(b) $P(X > \frac{3}{2}) = 1 - F(\frac{3}{2}) = \frac{7}{8}$

(c) $F(m) = \frac{1}{2}$: $m^3 = \frac{3^3}{2}$, so $m = 3 \times (0.5)^{\frac{1}{3}}$

$m = 2.38$

Answer: (b) $\frac{7}{8}$ (c) 2.38