Day 204 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean windspeed (knots) in the same month are:
Hurn: mean 11.5, standard deviation 1.2;
Beijing: mean 16.5, standard deviation 2.8.
Make two comparisons.

Means: Beijing has the higher mean (16.5 against 11.5), so on average the mean windspeed is greater at Beijing
Standard deviations: Beijing is larger at Beijing (2.8 against 1.2)
so the readings at Beijing are more variable and those at Hurn are more consistent
Answer: Beijing higher on average; Beijing more variable

2 Partial fractions: distinct linear factors (All boards)

Express $\frac{-7x + 11}{(2x - 5)(x + 4)}$ in partial fractions.

Let $\frac{-7x + 11}{(2x - 5)(x + 4)} \equiv \frac{A}{(2x - 5)} + \frac{B}{(x + 4)}$
$-7x + 11 \equiv A(x + 4) + B(2x - 5)$
Substitute $x = \frac{5}{2}$: $A = -1$
Substitute $x = -4$: $B = -3$
Answer: $\mathbf{\frac{-1}{(2x - 5)} + \frac{-3}{(x + 4)}}$

3 Parametric differentiation: tangent and normal (All boards)

A curve has parametric equations $x = 3t - 1$, $y = t^3$.

Find $\frac{dy}{dx}$ in terms of t, and the equation of the normal at the point where t = 2, in the form ax + by + c = 0 (integers).

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = t^2$
t = 2: point (5, 8), gradient 4
Normal gradient = $-1 \div 4 = -\frac{1}{4}$, so normal: $x + 4y - 37 = 0$
Answer: $\mathbf{\frac{dy}{dx} = t^2}$**; x + 4y − 37 = 0**

4 Partial fractions: improper fraction (All boards)

Express $\frac{2x^2 + x + 6}{(x + 2)(x - 1)}$ in the form $2 + \frac{A}{(x + 2)} + \frac{B}{(x - 1)}$, where A and B are constants to be found.

Divide: $2x^2 + x + 6 = 2(x^2 + x - 2) + (-x + 10)$
So the fraction is $2 + \frac{-x + 10}{(x + 2)(x - 1)}$
$A(x - 1) + B(x + 2) \equiv -x + 10$
$x = -2$ gives $A = -4$; $x = 1$ gives $B = 3$
Answer: A = −4, B = 3

5 Pulley, string goes slack (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (5 kg) and B (2 kg) hang on a light string over a smooth pulley, A being 2.5 m above the ground. Released from rest; A lands without rebounding. Find (a) the tension while both move, (b) the greatest height of B above its start. Take g = 9.8. Give answers to 3 s.f. if not exact.

A: $5g - T = 5a$; B: $T - 2g = 2a$
$a = 3g \div 7 = 4.2$, so $T = 2(g + a) = 28$
When A lands, B has risen 2.5 m with $v^2 = 2 \times 4.2 \times 2.5 = 21$
B then moves freely under gravity: extra height = $v^2 \div (2 \times 9.8) = \frac{15}{14}$
Total = $2.5 + \frac{15}{14}$
Answer: (a) 28 N (b) 3.57 m

Day 204 **A Level Maths: A Tier 10-a-day: Model Solutions**

6 Algebraic fractions (All boards)

Express $\frac{5}{(x+2)} - \frac{1}{(x+4)}$ as a single fraction in its simplest form.

Common denominator $(x + 2)(x + 4)$

Numerator: $5(x + 4) - 1(x + 2) = 4x + 18$

Answer: $\mathbf{\frac{4x + 18}{(x + 2)(x + 4)}}$

7 Continuous random variable: variance (CIE)

X has pdf $f(x) = \frac{6}{216}x(6 - x)$ for $0 \leq x \leq 6$, and 0 otherwise.

Find E(X), Var(X) and the standard deviation of X (3 s.f.).

$E(X) = \int_0^6 x\, f(x)\, dx = 3$

$E(X^2) = \int_0^6 x^2\, f(x)\, dx = \frac{54}{5}$

$Var(X) = \frac{54}{5} - (3)^2 = \frac{9}{5}$

Standard deviation $= \sqrt{\frac{9}{5}} = 1.34$

Answer: E(X) = 3, Var(X) = $\frac{9}{5}$, sd 1.34

8 R-form and solving a sin x + b cos x = c (All boards)

(a) Express $6 \cos x + 8 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give α to 1 d.p.

(b) Solve $6 \cos x + 8 \sin x = 5$ for $0° \leq x < 360°$, giving answers to 1 d.p.

$R^2 = 6^2 + 8^2 = 100$, so $R = 10$; $\tan \alpha = \frac{8}{6}$ gives $\alpha = 53.1°$

$10 \cos(x - 53.1°) = 5$, so $\cos(x - 53.1°) = \frac{1}{2}$

Principal value 60.0°, then use the symmetry of the cosine graph

$x = 113.1°, 353.1°$

Answer: (a) R = 10, α = 53.1° (b) x = 113.1°, 353.1°

9 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against x for $y = A e^{kx}$.

Find the exact values of A and k.

$\ln y = \ln A + kx$, a straight line with gradient k and intercept $\ln A$

Intercept $\ln A = 2$, so $A = e^2$

Gradient $k = \frac{8 - 2}{4 - 0} = \frac{3}{2}$

Answer: A = e^2, k = $\frac{3}{2}$

10 Closest approach of two particles (Edexcel, OCR, MEI)

At t = 0, P and Q have position vectors $(8\mathbf{i} - 8\mathbf{j})$ m and $(5\mathbf{i} + \mathbf{j})$ m and velocities $(-5\mathbf{i} + 4\mathbf{j})$ m s^{-1} and $(-3\mathbf{i} + 3\mathbf{j})$ m s^{-1}.

Find (a) the time when they are closest, (b) the least distance between them. Give answers to 3 s.f. if not exact.

$PQ = r_P - r_Q = (3\mathbf{i} - 9\mathbf{j}) + t(-2\mathbf{i} + \mathbf{j})$

$d^2 = (-2t + 3)^2 + (t - 9)^2 = 5t^2 - 30t + 90$

(a) $d(d^2)/dt = 0$: $10t + -30 = 0$, so $t = 3$

(b) At t = 3: $PQ = -3\mathbf{i} - 6\mathbf{j}$, so $d = \sqrt{45}$

Answer: (a) t = 3 s (b) 6.71 m