**Day 203** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Conservation of momentum (CIE)

Particles A (5 kg) and B (5 kg) move towards each other on a smooth horizontal line at 10 m $s^{-1}$ and 4 m $s^{-1}$. They collide directly, and A then moves in the opposite direction at 1 m $s^{-1}$. Find (a) the speed of B after the collision, (b) the magnitude of the impulse of A on B.

Take the initial direction of A as positive. Conservation of momentum:
$5\times10 + 5\times(-4) = 5\times(-1) + 5v$
$30 = -5 + 5v$, so $v = 7$
(b) Impulse on B = change in momentum of B $= 5(7 - (-4))$
$= 5 \times 11$

**Answer: (a) 7 m $s^{-1}$ (b) 55 N s**

## 2 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\sin^2\theta + 5\cos\theta - 4 = 0$
for $0° \le \theta \le 180°$.

Use $\sin^2\theta \equiv 1 - \cos^2\theta$: $2\cos^2\theta - 5\cos\theta + 2 = 0$

Solving the quadratic: $\cos\theta = \frac{1}{2}$ or $\cos\theta = 2$

$\cos\theta = \frac{1}{2}$: $\theta = 60°$

$\cos\theta = 2$ has no solutions ($|\cos\theta| \le 1$)

**Answer: $\theta = 60°$**

## 3 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 4 m $s^{-1}$ and accelerates uniformly. She travels 84 m in the next 12 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $84 = 4\times12 + \frac{1}{2}\times a\times12^2$, so $a = 0.5$
(b) $v = u + at = 4 + 0.5\times12 = 10$
(c) For the midpoint, $s = 42$: $v^2 = u^2 + 2as = 4^2 + 2\times0.5\times42$
$v^2 = 58$

**Answer: (a) 0.5 m $s^{-2}$ (b) 10 m $s^{-1}$ (c) 7.62 m $s^{-1}$**

## 4 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the mass (g), is normally distributed with mean 61 and standard deviation 5.
Find (a) P(X > 68.5) (b) P(68.5 < X < 76), each to 3 s.f.

(a) $z = \frac{68.5 - 61}{5} = 1.50$, $P(Z > 1.50) = 0.0668$

(b) $z_1 = 1.50$, $z_2 = \frac{76 - 61}{5} = 3.00$

$P(68.5 < X < 76) = \Phi(3.00) - \Phi(1.50) = 0.0655$

**Answer: (a) 0.0668 (b) 0.0655**

## 5 Double angle equations (All boards)

Solve $\cos2\theta - 5\cos\theta = -3$
for $0 \le \theta \le 2\pi$.

$\cos2\theta \equiv 2\cos^2\theta - 1$: $2\cos^2\theta - 5\cos\theta + 2 = 0$

Solve the quadratic: $\cos\theta = \frac{1}{2}$ or $\cos\theta = 2$

$\cos\theta = \frac{1}{2}$: $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$\cos\theta = 2$ is impossible

**Answer: $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$**

## 6 Factor theorem: roots with surds (All boards)

$f(x) = x^3 + 3x^2 - x - 6$.
(a) Show that $(x + 2)$ is a factor of $f(x)$.
(b) Solve $f(x) = 0$, giving surds in simplest form.

$f(-2) = -8 + 12 + 2 - 6 = 0$, so $(x + 2)$ is a factor.

$f(x) = (x + 2)(x^2 + x - 3)$

$x^2 + x - 3 = 0$ gives $x = \frac{-1 \pm \sqrt{13}}{2}$

**Answer: $x = -2$ or $x = \frac{-1 \pm \sqrt{13}}{2}$**

## 7 Interpreting a regression line (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows data and the regression line $y = 15 + 2x$.
(a) Interpret the gradient.
(b) Estimate $y$ when $x = 5$ and comment.

(a) On average, $y$ increases by 2 hundred sales for each extra 1 °C of $x$
(b) $y = 15 + 2 \times 5 = 25$
$x = 5$ is within the range of the data (2 to 12): interpolation, so reliable
**Answer: (a) y increases by 2 hundred sales per °C (b) 25, reliable**

## 8 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{5} - 2) = -1 - 4\sqrt{5}$, writing $x$ in the form $p + q\sqrt{5}$.

$x = \frac{-1 - 4\sqrt{5}}{\sqrt{5} - 2}$

Multiply top and bottom by $\sqrt{5} + 2$.
Numerator: $(-1 - 4\sqrt{5})(\sqrt{5} + 2) = -22 - 9\sqrt{5}$
Denominator: $5 - 4 = 1$
**Answer: $x = -22 - 9\sqrt{5}$**

## 9 Recurrence relations: limits (All boards)

$u_1 = 40$ and $u_{n+1} = \frac{1}{4} u_n + 15$.
(a) Find the limit $L$ of the sequence as $n \to \infty$.
(b) Find the least $n$ for which $|u_n - L| < 0.1$.

(a) $L = \frac{1}{4} L + 15$ gives $L = \frac{15}{1 - \frac{1}{4}} = 20$

(b) $u_n - L = (u_1 - L)\, r^{n-1} = 20 \times \left(\frac{1}{4}\right)^{n-1}$

$|20| \times \left(\frac{1}{4}\right)^{n-1} < \frac{1}{10}$

Check: $n = 4$ gives 0.31250, $n = 5$ gives 0.07812
**Answer: (a) $L = 20$ (b) $n = 5$**

## 10 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

$l_1$: $\mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$

$l_2$: $\mathbf{r} = \begin{pmatrix} 6 \\ -4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix}$

Determine whether $l_1$ and $l_2$ intersect. If they do, find the point of intersection.

Equate components: $-1 + 2\lambda = 6 - 2\mu$; $1 - 1\lambda = -4 + 3\mu$; $1 - 2\lambda = -1 - 2\mu$
Solving two of the equations gives a unique $(\lambda, \mu)$
which does not satisfy the third, so the lines do not meet
**Answer: The lines do not intersect (skew)**