Day 202 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Non-uniform rod and equal tensions (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB (length 2 m, mass 6 kg) has its centre of mass at G, AG = 0.75 m. It hangs horizontally from vertical strings at A and B. A particle of mass m kg is fixed at P, AP = 1.75 m, making the tensions equal. Find (a) m, (b) the tension. Take g = 9.8.

Equal tensions T: 2T = (6 + m)g, so T = ½(6 + m)g
Moments about A: T×2 = 6 g×0.75 + m g×1.75
½(6 + m)×2 = 4.5 + 1.75 m → 6 + 1 m = 4.5 + 1.75 m
m (1.75 − 1) = 6 − 4.5, so m = 2
(b) T = ½(6 + 2)×9.8
Answer: (a) m = 2 (b) 39.2 N

2 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of 80 people is taken from three groups.
9 of the sample are from Sales, which has 135 people.
(a) Find the total number of people.
(b) Admin has 210 people. How many are sampled?

Sales sampled fraction = $\frac{9}{135} = \frac{1}{15}$

Total = $80 \div \frac{1}{15} = 1200$

Admin: $210 \times \frac{1}{15} = 14$

Answer: (a) 1200 (b) 14

3 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix} -2 \\ 3 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix} -2 \\ 6 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $-2 + 2\lambda = -2 + 0\mu$; $3 + 3\lambda = 6 + 3\mu$; $-3 + 1\lambda = -1 + 0\mu$
Solving two of the equations gives a unique (λ, μ)
which does not satisfy the third, so the lines do not meet
Answer: The lines do not intersect (skew)

4 Independent and mutually exclusive events (All boards)

P(A) = 0.8 and P(A ∪ B) = 0.9.
(a) Given that A and B are independent, find P(B).
(b) Given instead that A and B are mutually exclusive, find P(B).
(c) In the case in (a), find P(A′ ∩ B).

(a) P(A ∪ B) = P(A) + P(B) − P(A)P(B), so 0.9 = 0.8 + P(B)(1 − 0.8)

P(B) = $\frac{0.9 - 0.8}{0.2} = 0.5$

(b) P(A ∩ B) = 0, so P(B) = 0.9 − 0.8 = 0.1
(c) P(A′ ∩ B) = P(A′)P(B) = 0.2 × 0.5 = 0.1
Answer: (a) 0.5 (b) 0.1 (c) 0.1

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5 Partial fractions: repeated factor (All boards)

Express $\frac{-5x^2 - 10x - 4}{(x+2)(x+1)^2}$ in the form $\frac{A}{(x+2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$, and find A, B and C.

$-5x^2 - 10x - 4 \equiv A(x+1)^2 + B(x+2)(x+1) + C(x+2)$
$x = -2$: $-4 = A(-1)^2$ so $A = -4$
$x = -1$: $1 = C(1)$ so $C = 1$
Coefficient of x^2: $-5 = A + B$ so $B = -1$
Answer: $A = -4$, $B = -1$, $C = 1$

6 Work-energy on a rough slope (CIE)

A 6 kg block is released from rest on a rough plane inclined at α, $\tan\alpha = \frac{3}{4}$, $\mu = 0.25$, and slides 8 m down a line of greatest slope. Use work and energy to find (a) the work done against friction, (b) its speed after 8 m. Take $g = 10$.

$R = 6g\cos\alpha = 48$ N, $F = \mu R = 12$ N
(a) Work done against friction $= F \times d = 12 \times 8$
(b) Loss of PE $= 6g \times 8\sin\alpha = 288$ J
Work-energy: $\frac{1}{2}\times 6 \times v^2 = 288 - 96 = 192$
$v^2 = 64$
Answer: (a) 96 J (b) 8 m s^{-1}

7 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $-2x^2 - 2xy + 3y^2 + 4y = 15$.
Show that P(3, −3) lies on the curve and find the equation of the tangent at P in the form $ax + by + c = 0$ with integer a, b, c.

Differentiate (product rule on the xy term): $(-4x - 2y) + (-2x + 6y + 4)\frac{dy}{dx} = 0$

At P(3, −3): $-6 + (-20)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{3}{10}$

Tangent: $y + 3 = (-\frac{3}{10})(x - 3)$, i.e. $3x + 10y + 21 = 0$

Answer: $3x + 10y + 21 = 0$

8 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{-1 + 2\sqrt{2}}{3 - 3\sqrt{2}}$ in the form $p + q\sqrt{2}$, where p and q are rational numbers.

Multiply top and bottom by $3 + 3\sqrt{2}$.
Denominator: $(3 - 3\sqrt{2})(3 + 3\sqrt{2}) = -9$
Numerator: $(-1 + 2\sqrt{2})(3 + 3\sqrt{2}) = 9 + 3\sqrt{2}$
So the expression is $\frac{9 + 3\sqrt{2}}{-9}$.

Answer: $-1 - \frac{1}{3}\sqrt{2}$

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that $\sqrt{7}$ is irrational.

Assume $\sqrt{7} = \frac{a}{b}$ with a, b integers with no common factor.

Then $7b^2 = a^2$, so a^2 is a multiple of 7, hence a is a multiple of 7.
Write $a = 7c$. Then $7b^2 = 49c^2$, so $b^2 = 7c^2$.
So b is also a multiple of 7, contradicting no common factor.
Answer: contradiction, so $\sqrt{7}$ is irrational

10 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2 x + 16\cot x + 8 = 0$
for $-180° < x < 180°$, $x \neq 0$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $4\cot^2 x + 16\cot x + 12 = 0$
$\cot x = -3$ or $\cot x = -1$

$\cot x = -3$ means $\tan x = -\frac{1}{3}$: $x = -18.4°, 161.6°$

$\cot x = -1$ means $\tan x = -1$: $x = -45.0°, 135.0°$

Answer: $x = -45.0°, -18.4°, 135.0°, 161.6°$