**Day 201** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Beam on two supports (AQA, Edexcel, OCR, MEI)

A uniform beam AB rests on supports at C and D, with a particle attached at A. Find the reactions at C and D. Take g = 9.8.

Moments about C: $R_D \times 3.5 = 40\,g \times 2 - 4\,g \times 3$

$R_D = 190$

Resolving vertically: $R_C + R_D = 44\,g = 431.2$

$R_C = 431.2 - 190$

**Answer: $R_C = 240.8$ N, $R_D = 190.4$ N**

## 2 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 375 items.
The 6th item chosen is number 86 and the 8th item chosen is number 116.
Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval = $(116 - 86) \div (8 - 6) = 15$
First item = $86 - 5 \times 15 = 11$
Sample size = $375 \div 15 = 25$
**Answer: Interval 15, first 11, size 25**

## 3 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

$|a| = 2$, $|b| = 6$ and the angle between a and b is 120°.
Find the exact values of $a \cdot b$ and $|a + b|$.

$a \cdot b = |a||b|\cos 120° = 2 \times 6 \times -\frac{1}{2} = -6$

$|a + b|^2 = |a|^2 + 2a \cdot b + |b|^2 = 4 + -12 + 36 = 28$

$|a + b| = 2\sqrt{7}$

**Answer: $a \cdot b = -6$, $|a + b| = 2\sqrt{7}$**

## 4 Completing the square (All boards)

$f(x) = 3x^2 + 18x + 21$.
(a) Write f(x) in the form $a(x + p)^2 + q$.
(b) State the minimum value of f(x) and the value of x at which it occurs.
(c) Solve $f(x) = 0$, giving surds in simplest form.

$f(x) = 3(x + 3)^2 - 6$
(b) minimum value $-6$ when $x = -3$
(c) $3(x + 3)^2 = 6$, so $(x + 3)^2 = 2$
$x + 3 = \pm\sqrt{2}$
**Answer: (a) $3(x + 3)^2 - 6$ (b) minimum $-6$ at $x = -3$ (c) $x = -3 \pm \sqrt{2}$**

## 5 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{5}{13}$ and $\cos B = -\frac{12}{13}$.
Find the exact values of sin(A+B) and tan(A−B).

$\cos A = \frac{12}{13}$ (A acute), $\sin B = \frac{5}{13}$ (B obtuse), so $\tan A = \frac{5}{12}$, $\tan B = -\frac{5}{12}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B = \frac{5}{13} \times (-\frac{12}{13}) + \frac{12}{13} \times \frac{5}{13} = 0$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\frac{5}{12} - (-\frac{5}{12})}{1 + \frac{5}{12} \times (-\frac{5}{12})} = \frac{120}{119}$$

**Answer: $\sin(A+B) = 0$, $\tan(A-B) = \frac{120}{119}$**

## 6 Inverse of a quadratic function (All boards)

$f(x) = x^2 - 8x + 19$, $x \geq 4$.

Find $f^{-1}(x)$ and state its domain.

$f(x) = (x - 4)^2 + 3$

Let $y = f(x)$: $(x - 4)^2 = y - 3$

$x \geq 4$, so take the positive root: $x = 4 + \sqrt{y - 3}$

**Answer: $f^{-1}(x) = 4 + \sqrt{x - 3}$, $x \geq 3$**

## 7 Conditional probability without replacement (All boards)

A bag has 6 green and 7 yellow counters. Two are taken at random without replacement.

Given that the counters are different colours, find the probability that the first is green.

$P(\text{green then yellow}) = \frac{6}{13} \times \frac{7}{12} = \frac{7}{26}$

$P(\text{yellow then green}) = \frac{7}{13} \times \frac{6}{12} = \frac{7}{26}$

$P(\text{different}) = \frac{7}{13}$

$P(\text{first green} \mid \text{different}) = \frac{\frac{7}{26}}{\frac{7}{13}}$

$= \frac{1}{2}$

**Answer: $\frac{1}{2}$**

## 8 Differential equations in context (All boards)

A drink at 82°C is left in a room at 22°C. Its temperature $\theta$ °C satisfies $\frac{d\theta}{dt} = -k(\theta - 22)$.

After 15 minutes the temperature is 72°C. Find k, then the time for it to reach 37°C, to 3 s.f.

$\int \frac{1}{\theta - 22}\, d\theta = \int -k\, dt$, so $\ln(\theta - 22) = -kt + C$

$t = 0$: $\theta - 22 = 60$, so $\theta - 22 = 60e^{-kt}$

$t = 15$: $50 = 60e^{-15k}$, so $k = \frac{\ln(\frac{6}{5})}{15} = 0.0122$

$15 = 60e^{-kt}$: $t = \frac{\ln(4)}{k} = 114$ minutes

**Answer: $k = 0.0122$, $t = 114$ minutes**

## 9 Range of horizontal force on a rough slope (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle is in equilibrium on a rough plane inclined at $\alpha$, $\tan\alpha = \frac{12}{5}$, $\mu = 0.4$. A horizontal force of P N, in the vertical plane through a line of greatest slope, pushes it towards the plane. Find the least and greatest values of P. Take g = 9.8.

$\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$, weight $W = 29.4$ N. Normal reaction $R = W\cos\alpha + P\sin\alpha$

Least P: about to slide down, friction $F = \mu R$ up the slope

$P\cos\alpha + \mu(W\cos\alpha + P\sin\alpha) = W\sin\alpha \rightarrow P = W(\sin\alpha - \mu\cos\alpha) \div (\cos\alpha + \mu\sin\alpha) = 30.0$

Greatest P: about to slide up, friction down the slope

$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) \rightarrow P = W(\sin\alpha + \mu\cos\alpha) \div (\cos\alpha - \mu\sin\alpha) = 2060$

**Answer: least 30 N, greatest 2058 N**

## 10 Discriminant: range of k (All boards)

The equation $3x^2 + (k - 5)x + (2k + 5) = 0$ has real roots.
Find the set of possible values of k.

Discriminant: $b^2 - 4ac = (k - 5)^2 - 4 \times 3(2k + 5)$
$= k^2 - 34k - 35$
Roots of the quadratic in k: $k = -1$ and $k = 35$
Required: $b^2 - 4ac \geq 0$
**Answer: $k \leq -1$ or $k \geq 35$**