Day 200 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_1^4 (4\sqrt{x} + \frac{2}{\sqrt{x}})\,dx$.

Integrate: $[\frac{8}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}}]$

Substitute the limits and subtract: $\frac{68}{3}$

Answer: $\mathbf{\frac{68}{3}}$

2 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $5^{3x+2} = 25^{2x-3}$.

Write both sides as powers of 5.

$(5^1)^{3x+2} = (5^2)^{2x-3}$

$3x + 2 = 2(2x - 3)$

$3x + 2 = 4x - 6$

$-x = -8$

Answer: x = 8

3 Normal approximation to the binomial (All boards)

X ~ B(200, 0.2). Use a normal approximation to estimate P(X ≥ 43) to 3 significant figures.

np = 40, npq = 32, so Y ~ N(40, 32)

With continuity correction: P(Y > 42.5)

$z = \frac{42.5 - 40}{5.66} = 0.442$, P(Z > 0.442) = 0.329

Answer: 0.329

4 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $2x^2 + 3xy + 2y^2 - y = 6$.

Show that P(−3, 2) lies on the curve and find the equation of the tangent at P in the form ax + by + c = 0 with integer a, b, c.

Differentiate (product rule on the xy term): $(4x + 3y) + (3x + 4y - 1)\frac{dy}{dx} = 0$

At P(−3, 2): $-6 + (-2)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -3$

Tangent: $y - 2 = (-3)(x + 3)$, i.e. $3x + y + 7 = 0$

Answer: 3x + y + 7 = 0

5 Transformations from a graph (All boards)

The diagram shows the curve y = f(x), with turning points A and B.

Find the coordinates of the turning points of

(a) y = f(x) + 3 (b) y = 2f(x)

(a) y = f(x) + 3: A → (−1, 4), B → (1, 0)

(b) y = 2f(x): A → (−1, 2), B → (1, −6)

Answer: (a) (−1, 4), (1, 0); (b) (−1, 2), (1, −6)

6 Double angle equations (All boards)

Solve $\cos 2\theta + 3\sin\theta = -1$
for $0 \le \theta \le 2\pi$.

$\cos 2\theta \equiv 1 - 2\sin^2\theta$: $2\sin^2\theta - 3\sin\theta - 2 = 0$

Solve the quadratic: $\sin\theta = -\frac{1}{2}$ or $\sin\theta = 2$

$\sin\theta = -\frac{1}{2}$: $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\sin\theta = 2$ is impossible

Answer: $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

7 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

20 values have $\Sigma x = 760$ and $\Sigma x^2 = 29200$.
One value was recorded as 44 but should have been 38.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 760 - 44 + 38 = 754$

$\Sigma x^2 = 29200 - 44^2 + 38^2 = 28708$

Mean $= \frac{754}{20} = 37.7$

sd $= \sqrt{\frac{28708}{20} - (37.7)^2} = 3.76$

Answer: mean 37.7, sd 3.76

8 Connected particles (rough table, string slack) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (2 kg) lies on a rough table, $\mu = 0.25$, joined by a light string over a smooth pulley to particle B (3 kg), 1.2 m above the floor. Released from rest; B lands without rebounding. Find (a) the acceleration of A before B lands, (b) the further distance A moves. Take $g = 9.8$.

B: $3g - T = 3a$; A: $T - 0.25\times 2g = 2a$
Adding: $(3 - 0.25\times 2)g = 5a$, so $a = 4.9$
Speed of B and A as B lands: $v^2 = 2\times 4.9\times 1.2 = 11.76$
After B lands, A decelerates at $\mu g = 0.25\times 9.8 = 2.45$
$0 = v^2 - 2(\mu g)s$, $s = 11.76 \div 4.9$

Answer: (a) 4.9 m s^{-2} (b) 2.4 m

9 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A(2, −7) and B(−10, 9).
Its centre lies on the line $y = x + 5$.
Find the equation of the circle.

Centre $(h, h + 5)$ is equidistant from A and B.
$(h - 2)^2 + (h + 12)^2 = (h + 10)^2 + (h - 4)^2$
Solving gives $h = -4$, so centre $(-4, 1)$
$r^2 = (6)^2 + (-8)^2 = 100$

Answer: $(x + 4)^2 + (y - 1)^2 = 100$

10 Closest approach of two particles (Edexcel, OCR, MEI)

At t = 0, P and Q have position vectors $(20\mathbf{i} - 7\mathbf{j})$ m and $(3\mathbf{i} - \mathbf{j})$ m and velocities $(-7\mathbf{i} - 5\mathbf{j})$ m s^{-1} and $(-3\mathbf{i} - 2\mathbf{j})$ m s^{-1}.
Find (a) the time when they are closest, (b) the least distance between them.

$\mathrm{PQ} = r_P - r_Q = (17\mathbf{i} - 6\mathbf{j}) + t(-4\mathbf{i} - 3\mathbf{j})$

$d^2 = (-4t + 17)^2 + (-3t - 6)^2 = 25t^2 - 100t + 325$
(a) $d(d^2)/dt = 0$: $50t + -100 = 0$, so $t = 2$
(b) At $t = 2$: $\mathrm{PQ} = 9\mathbf{i} - 12\mathbf{j}$, so $d = \sqrt{225}$

Answer: (a) t = 2 s (b) 15 m