Day 20 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean windspeed (knots) in the same month are:
Camborne: mean 15, standard deviation 5.1;
Leeming: mean 7.5, standard deviation 3.3.
Make two comparisons.

Means: Camborne has the higher mean (15 against 7.5), so on average the mean windspeed is greater at Camborne
Standard deviations: Camborne is larger at Camborne (5.1 against 3.3)
so the readings at Camborne are more variable and those at Leeming are more consistent
Answer: Camborne higher on average; Camborne more variable

2 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 88.2 m ahead. After a reaction time of 0.4 s she brakes with deceleration 2 m s^{-2} and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance = 0.4 v, braking distance = $v^2 \div (2\times2)$
$0.4\,v + v^2 \div 4 = 88.2$, so $v^2 + 1.6\,v - 352.8 = 0$
$(v - 18)(v + 19.6) = 0$, so $v = 18$
(b) Braking time = $18 \div 2 = 9$ s; total = 0.4 + 9
Answer: (a) 18 m s^{-1} (b) 9.4 s

3 Selections with combinations (OCR, MEI, Eduqas, CIE)

A group of 7 boys and 6 girls. A team of 3 is chosen at random.
Find the probability that the team has exactly 1 of the girls.

Total teams = C(13, 3) = 286
Favourable = C(6, 1) × C(7, 2) = 6 × 21 = 126

$$P = \frac{126}{286}$$

$$= \frac{63}{143}$$

Answer: $\mathbf{\frac{63}{143}}$

4 Chain rule and gradients (All boards)

$y = (4x - 6)^{-2}$.
Find $\frac{dy}{dx}$ and the gradient of the curve at $x = -1$.

Let $u = 4x - 6$, so $y = u^{-2}$, $\frac{du}{dx} = 4$, $\frac{dy}{du} = -2\,u^{-3}$

$$\frac{dy}{dx} = -8(4x - 6)^{-3}$$

At $x = -1$: $u = -10$, so gradient = $\frac{1}{125}$

Answer: $\mathbf{\frac{dy}{dx} = -8(4x - 6)^{-3}}$**; gradient =** $\mathbf{\frac{1}{125}}$

5 Arithmetic series: S_n formula (All boards)

The sum of the first n terms of an arithmetic series is $S_n = n^2 + n$.
(a) Find the first term and the common difference.
(b) Find u_{18}.

$u_1 = S_1 = 2$
$S_2 = 6$, so $u_2 = 4$
$d = u_2 - u_1 = 2$
$u_{18} = 2 + 17 \times 2$
Answer: (a) a = 2, d = 2 (b) 36

6 R-form and solving a sin x + b cos x = c (All boards)

(a) Express 12 cos x − 5 sin x in the form R cos(x + α), where R > 0 and 0° < α < 90°. Give α to 1 d.p.
(b) Solve 12 cos x − 5 sin x = −11 for 0° ≤ x < 360°, giving answers to 1 d.p.

$R^2 = 12^2 + 5^2 = 169$, so R = 13; $\tan\alpha = \frac{5}{12}$ gives α = 22.6°

$13\cos(x + 22.6°) = -11$, so $\cos(x + 22.6°) = \frac{-11}{13}$

Principal value 147.8°, then use the symmetry of the cosine graph
x = 125.2°, 189.6°
Answer: (a) R = 13, α = 22.6° (b) x = 125.2°, 189.6°

7 Recurrence relations: limits (All boards)

$u_1 = 0$ and $u_{n+1} = \frac{1}{3}u_n + 8$.

(a) Find the limit L of the sequence as n → ∞.
(b) Find the least n for which $|u_n - L| < 0.001$.

(a) $L = \frac{1}{3}L + 8$ gives $L = \frac{8}{1 - \frac{1}{3}} = 12$

(b) $u_n - L = (u_1 - L)\,r^{n-1} = (-12) \times \left(\frac{1}{3}\right)^{n-1}$

$|-12| \times \left(\frac{1}{3}\right)^{n-1} < \frac{1}{1000}$

Check: n = 9 gives 0.00183, n = 10 gives 0.00061
Answer: (a) L = 12 (b) n = 10

8 Rough incline: find the coefficient of friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle slides from rest down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{8}{15}$. It moves 19.6 m in 4 s.
Find (a) the coefficient of friction, (b) the speed after 4 s. Take g = 9.8.

$s = \frac{1}{2}at^2$: $19.6 = \frac{1}{2} \times a \times 4^2$, so a = 2.45
Along the slope: m g sin α − μR = m a, with R = m g cos α

$g(\sin\alpha - \mu\cos\alpha) = a$: $9.8\left(\frac{8}{17} - \frac{15}{17}\mu\right) = 2.45$

$\frac{8}{17} - \frac{15}{17}\mu = 0.25$, so μ = 0.25
v = at = 2.45×4
Answer: μ = 0.25; 9.8 m s^{-1}

9 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: 2 + 3λ = −1 − 2μ; 2 + 2λ = 0 − 3μ; −3 − 3λ = 0 + 2μ
Solving: λ = −1, μ = 0, and all three equations are satisfied
Point of intersection = (−1, 0, 0)
Answer: Intersect at (−1, 0, 0)

10 Arithmetic series: finding n (All boards)

An arithmetic series has first term 4 and common difference 1.
The sum of the first n terms is 85. Find n.

$\frac{n}{2}(2 \times 4 + (n - 1) \times 1) = 85$

$n^2 + 7n = 170$

$n^2 + 7n - 170 = 0$

$(n - 10)(n + 17) = 0$, so $n = 10$ or $n = -17$

n must be a positive integer, so $n = 10$

Answer: 10