Day 2 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show by counter-example that the statement "If ab is a multiple of 8, then a or b is a multiple of 8" is false.

Take a = 2 and b = 4:
ab = 8 is a multiple of 8,
but neither 2 nor 4 is a multiple of 8.
Answer: a = 2, b = 4

2 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2 x - 3\sin x - 3 = 0$
for $0 \leq x \leq 2\pi$.

Use $\cos^2 x \equiv 1 - \sin^2 x$: $2\sin^2 x + 3\sin x + 1 = 0$

Solving the quadratic: $\sin x = -\frac{1}{2}$ or $\sin x = -1$

$\sin x = -\frac{1}{2}$: $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\sin x = -1$: $x = \frac{3\pi}{2}$

Answer: $x = \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

3 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For n = 9 pairs of (x, y): $\Sigma x = 96$, $\Sigma y = 407$,
$\Sigma x^2 = 1376$, $\Sigma y^2 = 18897$, $\Sigma xy = 4642$.
Calculate the product moment correlation coefficient, and describe the correlation.

$S_{xx} = 1376 - \frac{96^2}{9} = 352$

$S_{yy} = 18897 - \frac{407^2}{9} = 492$

$S_{xy} = 4642 - \frac{96 \times 407}{9} = 301$

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = 0.723$

This is strong positive correlation
Answer: r = 0.723, positive correlation

4 Pulley, string goes slack (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (4 kg) and B (1 kg) hang on a light string over a smooth pulley, A being 2 m above the ground. Released from rest; A lands without rebounding. Find (a) the tension while both move, (b) the greatest height of B above its start. Take g = 9.8.

A: $4g - T = 4a$; B: $T - 1g = 1a$
$a = 3g \div 5 = 5.88$, so $T = 1(g + a) = 15.68$
When A lands, B has risen 2 m with $v^2 = 2 \times 5.88 \times 2 = 23.52$
B then moves freely under gravity: extra height = $v^2 \div (2 \times 9.8) = 1.2$
Total = 2 + 1.2
Answer: (a) 15.68 N (b) 3.2 m

5 Perpendicular bisector and area (All boards)

A is (4, −1) and B is (−6, −3).
(a) Find the equation of the perpendicular bisector of AB, in the form ax + by = c.
(b) The perpendicular bisector meets the x-axis at P. Find the area of triangle ABP.

Midpoint of AB = (−1, −2); gradient AB = $\frac{1}{5}$

Perpendicular gradient = −5
Equation: 5x + y = −7

y = 0: P = $(-\frac{7}{5}, 0)$

Area = ½ $|(-6-4)(0-(-1)) - (-\frac{7}{5}-4)(-3-(-1))|$

Answer: (a) 5x + y = −7 (b) $\frac{52}{5}$

6 Horizontal projection (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 10 m s^{-1} from a point 4.9 m above horizontal ground. Find (a) the horizontal distance to the landing point, (b) the speed and direction (angle below the horizontal, 1 d.p.) on landing. Take g = 9.8. Give answers to 3 s.f. if not exact.

Vertical: $4.9 = \frac{1}{2} \times 9.8 \times t^2$, so t = 1
(a) Horizontal distance = 10 × 1
(b) Vertical speed on landing = 9.8×1 = 9.8; horizontal speed = 10
Speed = $\sqrt{(10^2 + 9.8^2)} = 14.0$
tan θ = 9.8 ÷ 10, θ = 44.4°

Answer: (a) 10 m (b) 14.0 m s^{-1} at 44.4° below the horizontal

7 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $x^2 - y^2 = 34$.

Assume $x^2 - y^2 = (x - y)(x + y) = 34$.
x − y and x + y are both odd or both even (they differ by 2y).
Both odd: the product is odd, but 34 is even.
Both even: the product is a multiple of 4, but 34 is not.
Contradiction either way.

Answer: no such integers exist

8 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

The radius of a sphere increases at $\frac{1}{2}$ cm s^{-1}.

Find the rate of increase of its volume when r = 5 cm, in terms of π.

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} = 4\pi \times 5^2 \times \frac{1}{2} = 50\pi$

Answer: 50π cm^3/s

9 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bulbs are defective with probability 0.25, independently. A pack of 10 bulbs is rejected if it has at least 3 defective. From 8 such packs, find the probability that exactly 4 are rejected (3 s.f.).

Y ~ B(10, 0.25). P(rejected) = P(Y ≥ 3) = 1 − 0.526 = 0.474
W ~ B(8, 0.474) is the number rejected
$P(W = 4) = C(8, 4)(0.474)^4(1 - 0.474)^4 = 0.271$

Answer: 0.271

10 Quotient rule: stationary points (All boards)

Find the exact coordinates of the stationary point of

$y = \frac{\ln x}{x}$ for $x > 0$.

$$\frac{dy}{dx} = \frac{x \times \frac{1}{x} - \ln x \times 1}{x^2}$$

$$= \frac{1 - \ln x}{x^2}$$

$\frac{dy}{dx} = 0$: $\ln x = 1$, so $x = e$

$y = \frac{1}{1} \div e = e^{-1}$

Answer: (e, e^{-1})