**Day 199** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 2 m $s^{-1}$ and accelerates uniformly. She travels 35.2 m in the next 8 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $35.2 = 2\times8 + \frac{1}{2}\times a\times8^2$, so $a = 0.6$
(b) $v = u + at = 2 + 0.6\times8 = 6.8$
(c) For the midpoint, $s = 17.6$: $v^2 = u^2 + 2as = 2^2 + 2\times0.6\times17.6$
$v^2 = 25.12$
**Answer: (a) 0.6 m $s^{-2}$ (b) 6.8 m $s^{-1}$ (c) 5.01 m $s^{-1}$**

## 2 Sum of multiples in a range (All boards)

Find the sum of all the multiples of 6 that lie strictly between 53 and 600.

First multiple 54, last multiple 594, common difference 6.

Number of terms $n = \frac{594 - 54}{6} + 1 = 91$

$S = \frac{91}{2}(54 + 594) = 29484$

**Answer: 29484**

## 3 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2 x - \sin x - 1 = 0$
for $0° \le x \le 180°$.

Use $\cos^2 x \equiv 1 - \sin^2 x$: $2\sin^2 x + \sin x - 1 = 0$

Solving the quadratic: $\sin x = -1$ or $\sin x = \frac{1}{2}$

$\sin x = -1$ has no solutions ($|\sin x| \le 1$)

$\sin x = \frac{1}{2}$: $x = 30°, 150°$

**Answer: x = 30°, 150°**

## 4 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (t^3 - 5t)\mathbf{i} + (t^2 - 2t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when t = 4, (b) the value of t when P is moving parallel to $\mathbf{i}$. Give answers to 3 s.f. if not exact.

$\mathbf{v} = d\mathbf{r}/dt = (3t^2 - 5)\mathbf{i} + (2t - 2)\mathbf{j}$
(a) t = 4: $\mathbf{v} = 43\mathbf{i} + 6\mathbf{j}$
Speed $= \sqrt{(43^2 + 6^2)} = \sqrt{1885}$
(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $2t - 2 = 0$, so $t = 1$
**Answer: (a) 43.4 m $s^{-1}$ (b) t = 1**

## 5 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (2, 0, 2), (−1, −2, −3) and (−3, 3, 0).
Find angle ABC, in degrees to 1 d.p.

$\mathrm{BA} = \begin{pmatrix}3\\2\\5\end{pmatrix}$, $\mathrm{BC} = \begin{pmatrix}-2\\5\\3\end{pmatrix}$
$\mathrm{BA}\cdot\mathrm{BC} = 19$
$|\mathrm{BA}| = \sqrt{38}$, $|\mathrm{BC}| = \sqrt{38}$
$\cos \mathrm{ABC} = \frac{19}{\sqrt{38}\times\sqrt{38}} = 0.5000$, so angle ABC = 60.0°
**Answer: 60.0°**

## 6 Inverse of a quadratic function (All boards)

$f(x) = 3x^2 - 6x - 3$, $x \geq 1$.

Find $f^{-1}(x)$ and state its domain.

$f(x) = 3(x - 1)^2 - 6$

Let $y = f(x)$: $(x - 1)^2 = \frac{y + 6}{3}$

$x \geq 1$, so take the positive root: $x = 1 + \sqrt{\frac{y + 6}{3}}$

**Answer: $\mathbf{f^{-1}(x) = 1 + \sqrt{\frac{x + 6}{3}}, x \geq -6}$**

## 7 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

$l_1$: $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 12 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$

$l_2$: $\mathbf{r} = \begin{pmatrix} 7 \\ 7 \\ 10 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

Determine whether $l_1$ and $l_2$ intersect. If they do, find the point of intersection.

Equate components: $1 - 1\lambda = 7 + 2\mu$; $2 - 1\lambda = 7 + 1\mu$; $12 + 3\lambda = 10 + 2\mu$
Solving two of the equations gives a unique $(\lambda, \mu)$
which does not satisfy the third, so the lines do not meet
**Answer: The lines do not intersect (skew)**

## 8 Continuous random variable: cumulative distribution and median (CIE)

The random variable X has pdf $f(x) = \frac{4}{16}x^3$ for $0 \leq x \leq 2$, and 0 otherwise.

(a) Find the cumulative distribution function F(x) for $0 \leq x \leq 2$.
(b) Find P(X > 1). (c) Find the median of X to 3 s.f.

(a) $F(x) = \int_0^x f(t)\,dt = \frac{x^4}{16}$

(b) $P(X > 1) = 1 - F(1) = \frac{15}{16}$

(c) $F(m) = \frac{1}{2}$: $m^4 = \frac{2^4}{2}$, so $m = 2 \times (0.5)^{\frac{1}{4}}$

$m = 1.68$

**Answer: (b) $\frac{15}{16}$ (c) 1.68**

## 9 Bayes' theorem in context (All boards)

0.5% of people have a fault. The inspection test is positive for 90% of those who have it and for 5% of those who do not.

(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the fault (3 s.f.).

(a) $P(+) = 0.005 \times 0.9 + 0.995 \times 0.05$
$= 0.0045 + 0.04975 = 0.05425$

(b) $P(\text{has it} \mid +) = \frac{0.005 \times 0.9}{0.05425}$

$= 0.0829$

**Answer: (a) 0.05425 (b) 0.0829**

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## 10 Partial fractions: improper fraction (All boards)

Express $\frac{x^2 - 3x + 5}{(x - 3)(x - 2)}$ in the form $1 + \frac{A}{(x - 3)} + \frac{B}{(x - 2)}$, where A and B are constants to be found.

Divide: $x^2 - 3x + 5 = (x^2 - 5x + 6) + (2x - 1)$

So the fraction is $1 + \frac{2x - 1}{(x - 3)(x - 2)}$

$A(x - 2) + B(x - 3) \equiv 2x - 1$

$x = 3$ gives $A = 5$; $x = 2$ gives $B = -3$

**Answer: A = 5, B = −3**