Day 198 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

The volume of a spherical balloon increases at a constant rate of 100 cm^3s^{-1}.
Find the rates of increase of its radius and of its surface area when r = 4 cm.

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$. $\frac{dr}{dt} = \frac{dV}{dt \div dV/dr} = \frac{100}{4\pi r^2}$

r = 4: $\frac{dr}{dt} = \frac{25}{16\pi}$ cm s^{-1}

$A = 4\pi r^2$, so $\frac{dA}{dr} = 8\pi r$ and $\frac{dA}{dt} = 8\pi \times 4 \times \frac{25}{16\pi} = 50$ cm^2s^{-1}

Answer: $\frac{dr}{dt} = \frac{25}{16\pi}$ **cm/s,** $\frac{dA}{dt} = 50$ **cm^2/s**

2 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (-4\mathbf{i} - \mathbf{j})$ N, $F_2 = (12\mathbf{i} + 5\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 2 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$
i: $-4 + 12 + a = 0$, so $a = -8$
j: $-1 + 5 + b = 0$, so $b = -4$
(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(12^2 + 5^2)} = 13$ N
$a = 13 \div 2$
Answer: (a) a = −8, b = −4 (b) 6.5 m s^{-2}

3 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x - 5$ and $g(x) = x^2 - 2$.
Solve $gf(x) = 2$.

$gf(x) = g(x - 5) = (x - 5)^2 - 2$
$(x - 5)^2 - 2 = 2$
$x - 5 = \pm 2$
Answer: x = 3 or x = 7

4 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of 80 people is taken from three groups.
14 of the sample are from Sales, which has 70 people.
(a) Find the total number of people.
(b) Admin has 45 people. How many are sampled?

Sales sampled fraction = $\frac{14}{70} = \frac{1}{5}$

Total = $80 \div \frac{1}{5} = 400$

Admin: $45 \times \frac{1}{5} = 9$

Answer: (a) 400 (b) 9

5 Arithmetic series: S_n formula (All boards)

The sum of the first n terms of an arithmetic series is $S_n = n^2 + 3n$.
(a) Find the first term and the common difference.
(b) Find u_{29}.

$u_1 = S_1 = 4$
$S_2 = 10$, so $u_2 = 6$
$d = u_2 - u_1 = 2$
$u_{29} = 4 + 28 \times 2$
Answer: (a) a = 4, d = 2 (b) 60

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6 Implicit differentiation: products and cubes (All boards)

A curve has equation $2x^2y - y^3 - 3x = -1$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at (0, 1).

Differentiate: $4xy + 2x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} - 3 = 0$

$\frac{dy}{dx} = -\frac{4xy - 3}{2x^2 - 3y^2}$

At (0, 1): $\frac{dy}{dx} = -1$, so the normal has gradient 1

Answer: $\frac{dy}{dx} = -\frac{4xy - 3}{2x^2 - 3y^2}$; normal gradient 1

7 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(1, 3) and B(7, −5) are the ends of a diameter of a circle.

(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.

(b) Show that C(0, −4) lies on the circle.

Centre = midpoint of AB = (4, −1)

$r^2 = (-3)^2 + (4)^2 = 25$

$(x - 4)^2 + (y + 1)^2 = 25$: $x^2 + y^2 - 8x + 2y - 8 = 0$

(b) $CM^2 = (-4)^2 + (-3)^2 = 25 = r^2$

Answer: (a) $x^2 + y^2 - 8x + 2y - 8 = 0$

8 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 2125 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(150 + 100) × V

2125 = 125 V, so V = 17

(b) Deceleration = V ÷ 25 = 17 ÷ 25

Answer: (a) V = 17 (b) 0.68 m s^{-2}

9 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $-2 + 2\lambda = 0 + 1\mu$; $1 + 2\lambda = 3 - 2\mu$; $7 - 1\lambda = 5 - 2\mu$

Solving two of the equations gives a unique (λ, μ)

which does not satisfy the third, so the lines do not meet

Answer: The lines do not intersect (skew)

10 Histograms (All boards)

The histogram shows the waiting times, t (in minutes), of a sample.

(a) How many were in the class $30 \leq t < 50$?

(b) Estimate how many had $5 < t < 45$.

(a) Frequency = frequency density × width = 6 × 20 = 120

(b) 5 to 10: 3 × 5 = 15

whole bar 10 to 30: 20

30 to 45: 6 × 15 = 90

Total = 125

Answer: (a) 120 (b) 125