Day 197 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Partial fractions: distinct linear factors (All boards)

Express $\frac{-11x - 19}{(2x + 4)(x + 1)}$ in partial fractions.

Let $\frac{-11x - 19}{(2x + 4)(x + 1)} \equiv \frac{A}{(2x + 4)} + \frac{B}{(x + 1)}$

$-11x - 19 \equiv A(x + 1) + B(2x + 4)$

Substitute $x = -2$: $A = -3$

Substitute $x = -1$: $B = -4$

Answer: $\mathbf{\frac{-3}{(2x + 4)} + \frac{-4}{(x + 1)}}$

2 Change of sign to locate a root (All boards)

$f(x) = x^3 - x - 8$.

Show that the equation $f(x) = 0$ has a root α in the interval $[2, 3]$.

$f(x)$ is continuous on the interval

$f(2) = -2.00 < 0$

$f(3) = 16.0 > 0$

A change of sign, so a root lies in $[2, 3]$

Answer: Sign change: $f(2) = -2.00$, $f(3) = 16.0$

3 Final seconds of a fall (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a cliff. In the last 2 seconds before it hits the sea it falls 127.4 m. Find the height of the cliff. Take $g = 9.8$.

Let the total time of fall be t s. Then $s = 4.9t^2$ for the whole fall.

In the first $(t - 2)$ s the stone falls $4.9(t - 2)^2$.

$4.9t^2 - 4.9(t - 2)^2 = 127.4$

$4.9(4t - 4) = 127.4$, so $t = 7$

Height $= 4.9 \times 7^2 = 275.625$

Answer: 275.625 m

4 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$. The point P lies on AB with AP : PB = 3 : 2.

Find the position vector of P and the exact distance OP.

$\mathrm{OP} = \mathrm{OA} + \frac{3}{5}\mathrm{AB} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} + \frac{3}{5}\begin{pmatrix} 5 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

$\mathrm{OP} = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5} = \sqrt{5}$

Answer: $\mathbf{\begin{pmatrix} -2 \\ -1 \end{pmatrix}}$**, OP** $= \sqrt{5}$

5 Inverse of a rational function (All boards)

$f(x) = \frac{-x + 2}{3x + 1}$, $x \in \mathbb{R}$, $x \neq -\frac{1}{3}$.

Find $f^{-1}(x)$ and state its domain.

$y = \frac{-x + 2}{3x + 1}$: $y(3x + 1) = -x + 2$

$3xy + y = -x + 2$

$x(3y + 1) = -y + 2$

$x = \frac{-y + 2}{3y + 1}$, then swap x and y

Answer: $\mathbf{f^{-1}(x) = \frac{-x + 2}{3x + 1}}$**,** $\mathbf{x \neq -\frac{1}{3}}$

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6 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (3 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (5 kg), hanging 1.5 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: $5g - T = 5a$
A: $T = 3a$
Adding: $5g = 8a$, so $a = 6.125$
(a) $T = 3 \times 6.125 = 18.375$
(b) $v^2 = 0 + 2 \times 6.125 \times 1.5 = 18.375$
Answer: (a) 18.4 N (b) 4.29 m s^{-1}

7 Cubic inequalities (All boards)

Given that $(x + 3)$ is a factor of $f(x) = x^3 - 3x^2 - 10x + 24$, solve $f(x) < 0$.

Factorise: $f(x) = (x + 3)(x - 2)(x - 4)$
Roots at $x = -3, 2, 4$
Sketch the cubic (positive x^3 coefficient: negative, positive, negative, positive)
Read off where $f(x) < 0$.
Answer: $x < -3$ or $2 < x < 4$

8 Continuous random variable: variance (CIE)

X has pdf $f(x) = \frac{3}{64}x^2$ for $0 \le x \le 4$, and 0 otherwise.

Find E(X), Var(X) and the standard deviation of X (3 s.f.).

$E(X) = \int_0^4 x\, f(x)\, dx = 3$

$E(X^2) = \int_0^4 x^2\, f(x)\, dx = \frac{48}{5}$

$\text{Var}(X) = \frac{48}{5} - (3)^2 = \frac{3}{5}$

Standard deviation $= \sqrt{\frac{3}{5}} = 0.775$

Answer: E(X) = 3, Var(X) = $\frac{3}{5}$, sd 0.775

9 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $4x^2 + 2xy + y^2 = 48$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{8x + 2y}{2x + 2y}$
Horizontal tangent: $8x + 2y = 0$, so $y = -4x$
Substitute: $12x^2 = 48$, so $x^2 = 4$ and $x = \pm 2$
Points: $(2, -8)$ and $(-2, 8)$
Answer: (2, −8) and (−2, 8)

10 Hypothesis test for a normal mean (All boards)

The mean mass of a sack (kg) is normally distributed with standard deviation 10; it was previously 73.
A random sample of 49 has mean 75.
Test, at the 5% level, whether the mean has increased.

$H_0: \mu = 73$, $H_1: \mu > 73$

$\bar{X} \sim N(73, \frac{10^2}{49})$, $z = \frac{75 - 73}{\frac{10}{7}} = 1.40$

p-value = 0.0808 > 0.05
Do not reject H_0: no evidence that the mean has increased.
Answer: z = 1.40, p = 0.0808; do not reject H_0