Day 196 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Collinear points and ratios (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A, B and C have position vectors $\begin{pmatrix}5\\-4\\-3\end{pmatrix}$, $\begin{pmatrix}5\\-3\\0\end{pmatrix}$ and $\begin{pmatrix}5\\-1\\k\end{pmatrix}$ relative to O, where k is a constant. Given that A, B and C are collinear, find k and the ratio AB : BC.

AB = $\begin{pmatrix}0\\1\\3\end{pmatrix}$, and AC = $\begin{pmatrix}0\\3\\k+3\end{pmatrix}$

AC = λAB: component 3 gives 9 = 3 λ, so λ = 3

Hence k = −3 + 3 × 3 = 6

AC = 3 AB, so BC = 2 AB: AB : BC = 1 : 2

Answer: k = 6, AB : BC = 1 : 2

2 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show by counter-example that the statement "n! + 1 is prime for all positive integers n" is false.

Try n = 5:

$5! + 1 = 121 = 11^2$

This is not prime.

Answer: n = 5 gives 121 = 11^2

3 Projectile from and to the ground (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 49 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{4}{3}$, and lands at A. Find (a) the time of flight, (b) the greatest height, (c) OA. Take g = 9.8.

Horizontal component = 49×0.6 = 29.4; vertical component = 49×0.8 = 39.2

(a) Vertical: $0 = 39.2\,t - 4.9t^2$, so t = 8

(b) At the top: $0 = 39.2^2 - 2\times9.8\times H$, so H = 78.4

(c) OA = 29.4 × 8

Answer: (a) 8 s (b) 78.4 m (c) 235.2 m

4 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(−1, −1) and B(−7, 7) are the ends of a diameter of a circle.

(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.

(b) Show that C(−7, −1) lies on the circle.

Centre = midpoint of AB = (−4, 3)

$r^2 = (3)^2 + (-4)^2 = 25$

$(x + 4)^2 + (y - 3)^2 = 25$: $x^2 + y^2 + 8x - 6y = 0$

(b) $CM^2 = (-3)^2 + (-4)^2 = 25 = r^2$

Answer: (a) $x^2 + y^2 + 8x - 6y = 0$

5 Modelling with a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

At one location, the daily maximum gust is modelled by N(20.6, 6^2).

(a) Find the probability that on a randomly chosen day the value exceeds 30 knots.

(b) Estimate the number of such days in a period of 92 days.

(a) $z = \frac{30 - 20.6}{6} = 1.57$

P(Z > 1.57) = 0.0586

(b) 92 × 0.0586 = 5.39 ≈ 5

Answer: (a) 0.0586 (b) 5 days

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6 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 3t^2 + 1$, $y = -t(t^2 - 9)$.
Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 9$, so $t = 0, 3, -3$
$t = 0$: $x = 1$
$t = \pm 3$: $x = 28$
Answer: (1, 0) and (28, 0)

7 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bulbs are defective with probability 0.25, independently. A pack of 7 bulbs is rejected if it has at least 3 defective.
From 10 such packs, find the probability that exactly 2 are rejected (3 s.f.).

$Y \sim B(7, 0.25)$. $P(\text{rejected}) = P(Y \geq 3) = 1 - 0.756 = 0.244$
$W \sim B(10, 0.244)$ is the number rejected
$P(W = 2) = C(10, 2)(0.244)^2(1 - 0.244)^8 = 0.286$
Answer: 0.286

8 Double angle equations (All boards)

Solve $\cos 2x + 2\cos x = -1$
for $0 \leq x \leq 2\pi$.

$\cos 2x \equiv 2\cos^2 x - 1$: $2\cos^2 x + 2\cos x = 0$
Solve the quadratic: $\cos x = 0$ or $\cos x = -1$
$\cos x = 0$: $x = \frac{\pi}{2}, \frac{3\pi}{2}$
$\cos x = -1$: $x = \pi$
Answer: $x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

9 Greatest and least values using R-form (All boards)

$g(x) = \frac{1}{18 + 6\sin x + 8\cos x}$
(a) Write $6\sin x + 8\cos x$ in the form $R\sin(x + \alpha)$, with α acute.
(b) Find the greatest and least values of $g(x)$ and the smallest positive x at which the greatest value occurs (1 d.p.).

$R = \sqrt{6^2 + 8^2} = 10$, $\alpha = \tan^{-1}\frac{8}{6} = 53.1°$
Denominator $= 18 + 10\sin(x + 53.1°)$, ranging from 8 to 28
Greatest $g = \frac{1}{8}$ (denominator least), least $g = \frac{1}{28}$
Least denominator when $\sin(x + 53.1°) = -1$: $x + 53.1° = 270°$
$x = 216.9°$
Answer: $R = 10$, $\alpha = 53.1°$; greatest $\frac{1}{8}$, least $\frac{1}{28}$; $x = 216.9°$

10 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (2t^3 - 5t)\mathbf{i} + (3t^2 - 6t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when $t = 3$, (b) the value of t when P is moving parallel to $\mathbf{i}$. Give answers to 3 s.f. if not exact.

$\mathbf{v} = d\mathbf{r}/dt = (6t^2 - 5)\mathbf{i} + (6t - 6)\mathbf{j}$
(a) $t = 3$: $\mathbf{v} = 49\mathbf{i} + 12\mathbf{j}$
Speed $= \sqrt{(49^2 + 12^2)} = \sqrt{2545}$
(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $6t - 6 = 0$, so $t = 1$
Answer: (a) 50.4 m s^{-1} (b) $t = 1$