Day 195 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate $\int_0^2 2(\sqrt{x^3 + 1})\,dx$, giving your answer to 3 s.f.

State, with a reason, whether your estimate is an overestimate or an underestimate.

h = $\frac{1}{2}$; y-values at x = 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2: 2.0000, 2.1213, 2.8284, 4.1833, 6.0000

T = $\frac{1}{4}$[2.0000 + 6.0000 + 2(2.1213 + 2.8284 + 4.1833)]

T = 6.57

The curve is convex (y'' > 0) on this interval, so the chords lie above the curve: overestimate

Answer: 6.57, an overestimate

2 Partial fractions: distinct linear factors (All boards)

Express $\frac{15x + 12}{(2x + 4)(x - 4)}$ in partial fractions.

Let $\frac{15x + 12}{(2x + 4)(x - 4)} \equiv \frac{A}{(2x + 4)} + \frac{B}{(x - 4)}$

$15x + 12 \equiv A(x - 4) + B(2x + 4)$

Substitute $x = -2$: $A = 3$

Substitute $x = 4$: $B = 6$

Answer: $\frac{3}{(2x + 4)} + \frac{6}{(x - 4)}$

3 Interpolation for median and percentiles (All boards)

The table summarises the length, l mm, of leaves of 45 items.

Use linear interpolation to estimate the 90th percentile.

Position = 0.9 × 45 = 40.5

Cumulative frequency before the class $60 \leq l < 70$ is 27

Estimate = $60 + \frac{40.5 - 27}{18} \times 10$

= 67.5

Answer: 67.5

4 Double and triple angle values (All boards)

Given that $\cos\theta = \frac{3}{4}$, find the exact value of $\cos 3\theta$.

$\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$

$= (2\cos^2\theta - 1)\cos\theta - 2\sin^2\theta \cos\theta = 4\cos^3\theta - 3\cos\theta$

$= 4 \times \left(\frac{3}{4}\right)^3 - 3 \times \frac{3}{4} = -\frac{9}{16}$

Answer: $-\frac{9}{16}$

5 Binomial mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X has a binomial distribution with mean 28 and variance 14.

(a) Find n and p.

(b) Find P(X = 30) to 3 significant figures.

(a) np = 28 and npq = 14

$q = \frac{npq}{np} = \frac{14}{28} = 0.5$, so p = 0.5

$n = \frac{28}{0.5} = 56$

(b) $P(X = 30) = C(56, 30)(0.5)^{30}(0.5)^{26} = 0.0922$

Answer: (a) n = 56, p = 0.5 (b) 0.0922

6 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$5\sin 3\theta + 3\cos 2\theta = 6$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin 3\theta \approx 3\theta$, $\cos 2\theta \approx 1 - \frac{4}{2}\theta^2$

$5 \times 3\theta + 3(1 - \frac{4}{2}\theta^2) = 6$

Multiply by 2 and rearrange: $12\theta^2 - 30\theta + 6 = 0$

Divide through by 6: $2\theta^2 - 5\theta + 1 = 0$

$\theta = \frac{5 \pm \sqrt{17}}{4}$, so $\theta = 0.219$

Answer: a = 2, b = −5, c = 1; θ ≈ 0.219

7 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has constant acceleration $(\mathbf{i} - 3\mathbf{j})$ m s^{-2}. When t = 0, P is at $(3\mathbf{i} - 3\mathbf{j})$ m moving with velocity $(-2\mathbf{i} - \mathbf{j})$ m s^{-1}. Find (a) when P moves parallel to **j**, (b) its position vector then.

$\mathbf{v} = \mathbf{u} + \mathbf{at} = (-2\mathbf{i} - \mathbf{j}) + t(\mathbf{i} - 3\mathbf{j})$
(a) Parallel to **j**: the **i** component is zero: $-2 + t = 0$, so $t = 2$
(b) $\mathbf{r} = \mathbf{r}_0 + \mathbf{ut} + \frac{1}{2}\mathbf{at}^2$
$= (3\mathbf{i} - 3\mathbf{j}) + 2(-2\mathbf{i} - \mathbf{j}) + 2(\mathbf{i} - 3\mathbf{j})$
$= \mathbf{i} - 11\mathbf{j}$
Answer: (a) t = 2 (b) r = (i − 11j) m

8 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is −36 and the coefficient of x^2 is 594. Find n and k.

$nk = -36$ and $\frac{n(n-1)}{2}k^2 = 594$

$(n - 1)k = \frac{2 \times 594}{-36} = -33$ (dividing the second equation by the first)

$n\,k - (n - 1)\,k = k = -36 - (-33) = -3$

$n = \frac{-36}{-3} = 12$

Answer: n = 12, k = −3

9 Transformations from a graph (All boards)

The diagram shows the curve y = f(x), with turning points A and B.
Find the coordinates of the turning points of
(a) y = f(x − 1) (b) y = −f(x)

(a) y = f(x − 1): A → (0, 4), B → (2, −4)
(b) y = −f(x): A → (−1, −4), B → (1, 4)
Answer: (a) (0, 4), (2, −4); (b) (−1, −4), (1, 4)

10 Closest approach of two particles (Edexcel, OCR, MEI)

At t = 0, P and Q have position vectors $(-7\mathbf{i} + 10\mathbf{j})$ m and $(-\mathbf{i})$ m and velocities $(4\mathbf{i} - 4\mathbf{j})$ m s^{-1} and $(2\mathbf{i} - 2\mathbf{j})$ m s^{-1}. Find (a) the time when they are closest, (b) the least distance between them. Give answers to 3 s.f. if not exact.

$\mathbf{PQ} = \mathbf{r}_P - \mathbf{r}_Q = (-6\mathbf{i} + 10\mathbf{j}) + t(2\mathbf{i} - 2\mathbf{j})$

$d^2 = (2t - 6)^2 + (-2t + 10)^2 = 8t^2 - 64t + 136$
(a) $d(d^2)/dt = 0$: $16\,t + -64 = 0$, so $t = 4$
(b) At t = 4: $\mathbf{PQ} = 2\mathbf{i} + 2\mathbf{j}$, so $d = \sqrt{8}$
Answer: (a) t = 4 s (b) 2.83 m