Day 194 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

$|a| = 2$, $|b| = 2$ and the angle between a and b is 30°.
Find the exact values of $a \cdot b$ and $|a + b|$.

$a \cdot b = |a||b|\cos 30° = 2 \times 2 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

$|a + b|^2 = |a|^2 + 2a \cdot b + |b|^2 = 4 + 4\sqrt{3} + 4 = 4\sqrt{3} + 8$

$|a + b| = \sqrt{4\sqrt{3} + 8}$

Answer: $a \cdot b = 2\sqrt{3}$, $|a + b| = \sqrt{4\sqrt{3} + 8}$

2 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(16, −4) and B(−10, −4) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(−9, 1) lies on the circle.

Centre = midpoint of AB = (3, −4)

$r^2 = (13)^2 + (0)^2 = 169$

$(x - 3)^2 + (y + 4)^2 = 169$: $x^2 + y^2 - 6x + 8y - 144 = 0$

(b) $CM^2 = (-12)^2 + (5)^2 = 169 = r^2$

Answer: (a) $x^2 + y^2 - 6x + 8y - 144 = 0$

3 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\sin^2\theta - 3\cos\theta - 3 = 0$
for $-180° \le \theta \le 180°$.

Use $\sin^2\theta \equiv 1 - \cos^2\theta$: $2\cos^2\theta + 3\cos\theta + 1 = 0$

Solving the quadratic: $\cos\theta = -1$ or $\cos\theta = -\frac{1}{2}$

$\cos\theta = -1$: $\theta = -180°, 180°$

$\cos\theta = -\frac{1}{2}$: $\theta = -120°, 120°$

Answer: $\theta = -180°, -120°, 120°, 180°$

4 Independent and mutually exclusive events (All boards)

P(A) = 0.6 and $P(A \cup B) = 0.68$.
(a) Given that A and B are independent, find P(B).
(b) Given instead that A and B are mutually exclusive, find P(B).
(c) In the case in (a), find $P(A' \cap B)$.

(a) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, so $0.68 = 0.6 + P(B)(1 - 0.6)$

$P(B) = \frac{0.68 - 0.6}{0.4} = 0.2$

(b) $P(A \cap B) = 0$, so $P(B) = 0.68 - 0.6 = 0.08$

(c) $P(A' \cap B) = P(A')P(B) = 0.4 \times 0.2 = 0.08$

Answer: (a) 0.2 (b) 0.08 (c) 0.08

5 Partial fractions and integration (All boards)

(a) Express $\frac{-2x - 7}{(x + 2)^2}$ in the form $\frac{A}{x + 2} + \frac{B}{(x + 2)^2}$.

(b) Hence find the exact value of $\int_0^4 \frac{-2x - 7}{(x + 2)^2}\,dx$.

$-2x - 7 \equiv A(x + 2) + B$, so $A = -2$ and $B = -3$

$I = \left[-2\ln(x + 2) - \frac{-3}{x + 2}\right]_0^4$

$= -2\ln\frac{6}{2} + -3(\frac{1}{2} - \frac{1}{6})$

$= -2\ln 3 - 1$

Answer: (a) $A = -2$, $B = -3$; (b) $-2\ln 3 - 1$

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6 Rough incline: find the coefficient of friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle slides from rest down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{7}{24}$. It moves 22.54 m in 5 s. Find (a) the coefficient of friction, (b) the speed after 5 s. Take g = 9.8.

$s = \frac{1}{2}at^2$: $22.54 = \frac{1}{2}\times a\times 5^2$, so $a = 1.8032$

Along the slope: $mg\sin\alpha - \mu R = ma$, with $R = mg\cos\alpha$

$g(\sin\alpha - \mu\cos\alpha) = a$: $9.8(0.28 - 0.96\mu) = 1.8032$

$0.28 - 0.96\mu = 0.184$, so $\mu = 0.1$

$v = at = 1.8032\times 5$

Answer: $\mu = 0.1$; 9.016 m s^{-1}

7 Poisson over different intervals (CIE)

Buses arrive at a stop at random at an average of 8 per hour.
Find the probability that in a 120-minute period there are (a) zero events (b) at least 1.
(c) Find the probability that at least one occurs in each of two separate 120-minute periods.

$\lambda = 8\times\frac{120}{60} = 16$, $X \sim Po(16)$

(a) $P(X = 0) = e^{-16} = 0.000000113$

(b) $P(X \geq 1) = 1 - P(X \leq 0) = 1.00$

(c) $(1 - 0.000000113)^2 = 1.00$

Answer: (a) 0.000000113 (b) 1.00 (c) 1.00

8 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $3x - 4y = 22$ meets the circle $(x + 4)^2 + (y + 4)^2 = 36$ at the points A and B. Find the exact length of AB.

Centre $(-4, -4)$, radius 6. Distance from centre to line:

$d = \frac{|-18|}{5} = \frac{18}{5}$

$\text{Half-chord}^2 = 36 - (\frac{18}{5})^2 = \frac{576}{25}$

$AB = 2\sqrt{\frac{576}{25}}$

Answer: $\frac{48}{5}$

9 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 14 and the coefficient of x^2 is 84. Find n and k.

$nk = 14$ and $\frac{n(n-1)}{2}k^2 = 84$

$(n - 1)k = \frac{2\times 84}{14} = 12$ (dividing the second equation by the first)

$n\,k - (n - 1)\,k = k = 14 - 12 = 2$

$n = \frac{14}{2} = 7$

Answer: n = 7, k = 2

10 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$, and passes through A, 10 m horizontally from O and 19.1 m above the ground. Find the speed of projection. Take g = 9.8.

Let the speed be U. Horizontal: $10 = U\cos\alpha \times t$, so $t = 10 \div (\frac{5}{13}\ U)$

Vertical: $19.1 = U\sin\alpha \times t - 4.9t^2$

Eliminate t: $y = x\tan\alpha - 4.9x^2 \div (U^2\cos^2\alpha)$

$19.1 = 24 - 4.9\times10^2 \div (\frac{25}{169}\ U^2)$

$U^2 = 3312.4 \div 4.9 = 676$, $U = 26$

Answer: 26 m s^{-1}