Day 193 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{-4-\sqrt{6}}{1+\sqrt{6}}$ in the form $p + q\sqrt{6}$, where p and q are rational numbers.

Multiply top and bottom by $1 - \sqrt{6}$.
Denominator: $(1 + \sqrt{6})(1 - \sqrt{6}) = -5$
Numerator: $(-4 - \sqrt{6})(1 - \sqrt{6}) = 2 + 3\sqrt{6}$
So the expression is $\frac{2+3\sqrt{6}}{-5}$.

Answer: $\mathbf{-\frac{2}{5} - \frac{3}{5}\sqrt{6}}$

2 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against x for $y = A\,b^x$.
Find the exact values of A and b.

ln y = ln A + x ln b, a straight line with gradient ln b and intercept ln A
Intercept ln A = 1, so $A = e^1$
Gradient ln b = $\frac{0-1}{2-0} = -\frac{1}{2}$, so $b = e^{-\frac{1}{2}}$

Answer: $\mathbf{A = e^1, b = e^{-\frac{1}{2}}}$

3 Line meets curve: length of chord (All boards)

The line $y = 2x - 2$ meets the curve $y = x^2 - 4x + 6$ at A and B.
(a) Find the coordinates of A and B.
(b) Find the exact length AB.

$x^2 - 4x + 6 = 2x - 2$ gives $x^2 - 6x + 8 = 0$
$(x - 2)(x - 4) = 0$: x = 2 or 4
A(2, 2), B(4, 6)
$AB^2 = 2^2 + 4^2 = 20$

Answer: **A(2, 2), B(4, 6); AB** $= \mathbf{2\sqrt{5}}$

4 Outliers (AQA, Edexcel, OCR, MEI, Eduqas)

A data set has mean 53 and standard deviation 5.
A value is treated as an outlier if it is more than 2 standard deviations from the mean.
Which of 42, 47, 61, 65 are outliers?

2 standard deviations = 2 × 5 = 10
Lower limit = 53 − 10 = 43
Upper limit = 53 + 10 = 63
Outliers are values below 43 or above 63

Answer: 42, 65

5 Double angle equations (All boards)

Solve $\cos 2\theta + \sin\theta = 1$
for $0 \le \theta \le 2\pi$.

$\cos 2\theta \equiv 1 - 2\sin^2\theta$: $2\sin^2\theta - \sin\theta = 0$
Solve the quadratic: $\sin\theta = \frac{1}{2}$ or $\sin\theta = 0$
$\sin\theta = \frac{1}{2}$: $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$
$\sin\theta = 0$: $\theta = 0, \pi, 2\pi$

Answer: $\boldsymbol{\theta = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, 2\pi}$

6 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (2t^3 - 5t)\mathbf{i} + (t^2 - 4t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when t = 3, (b) the value of t when P is moving parallel to $\mathbf{i}$. Give answers to 3 s.f. if not exact.

$\mathbf{v} = d\mathbf{r}/dt = (6t^2 - 5)\mathbf{i} + (2t - 4)\mathbf{j}$
(a) t = 3: $\mathbf{v} = 49\mathbf{i} + 2\mathbf{j}$
Speed = $\sqrt{(49^2 + 2^2)} = \sqrt{2405}$
(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $2t - 4 = 0$, so t = 2
Answer: (a) 49.0 m s^{-1} (b) t = 2

7 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $2x^2 - 4xy + 2y^2 - 3x + 4y = 30$.
Show that P(0, 3) lies on the curve and find the equation of the tangent at P in the form ax + by + c = 0 with integer a, b, c.

Differentiate (product rule on the xy term): $(4x - 4y - 3) + (-4x + 4y + 4)\frac{dy}{dx} = 0$

At P(0, 3): $-15 + (16)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = \frac{15}{16}$

Tangent: $y - 3 = \frac{15}{16}(x + 0)$, i.e. $15x - 16y + 48 = 0$
Answer: 15x − 16y + 48 = 0

8 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (13 m, 20 kg) rests with A on rough ground, $\mu = \frac{1}{3}$, and B against a smooth wall, B being 12 m above the ground. A man of mass 75 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(13^2 - 12^2)} = 5$ m from the wall

Vertical: R = (20 + 75)g. Limiting friction: $F = \mu R = \frac{1}{3}(20 + 75)g$

Horizontal: S = F (wall reaction S)
Moments about A (man at distance x along ladder, horizontal distance x×5/13):
S×12 = 20 g×2.5 + 75 g×(5 x ÷ 13)
$\frac{95}{3} \times 12 = 50 + \frac{375}{13}$ x (g cancels), so x = 11.44
Answer: 11.44 m

9 Sum to infinity: two possible ratios (All boards)

The second term of a geometric series is 12 and its sum to infinity is 50.
Find the two possible values of the first term.

$ar = 12$ and $\frac{a}{(1 - r)} = 50$
$a = 50(1 - r)$, so $50(1 - r)\, r = 12$
$50r^2 - 50r + 12 = 0$: $r = \frac{2}{5}$ or $r = \frac{3}{5}$
$a = 50(1 - r) = 30$ or 20
Answer: a = 20 or a = 30

10 Normal approximation to the Poisson (CIE)

X ~ Po(45). Use a suitable approximation to find $P(X \leq 50)$, to 3 significant figures.

$\lambda > 15$, so Y ~ N(45, 45) approximately
Continuity correction: P(Y < 50.5)
z = 0.820, so probability = 0.794
Answer: 0.794