Day 192 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Differentiating a^x (All boards)

$y = 2^x$.
Find the exact gradient of the curve at x = 1 and the equation of the tangent there.

$\frac{dy}{dx} = 1 \times 1 \ln 2 \times 2^x = \ln 2 \times 2^x$
At x = 1: y = 2 and gradient = 2 ln 2
Tangent: $y = (2 \ln 2)x + 2 - 2 \ln 2$
Answer: gradient 2 ln 2; $y = (2 \ln 2)x + 2 - 2 \ln 2$

2 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 3 kg is acted on by forces $F_1 = (2\mathbf{i} - 15\mathbf{j})$ N and F_2. Its acceleration is $(-\mathbf{i} - \mathbf{j})$ m s^{-2}. Find (a) F_2, (b) the angle between F_2 and $\mathbf{i}$, to 1 d.p.

F = ma: $F_1 + F_2 = 3(-\mathbf{i} - \mathbf{j}) = -3\mathbf{i} - 3\mathbf{j}$
(a) $F_2 = (-3\mathbf{i} - 3\mathbf{j}) - (2\mathbf{i} - 15\mathbf{j}) = -5\mathbf{i} + 12\mathbf{j}$
(b) $|F_2| = \sqrt{(-5^2 + 12^2)} = 13$
cos θ = −5 ÷ 13, so θ = 112.6°
Answer: (a) $F_2 = (-5\mathbf{i} + 12\mathbf{j})$ N (b) 112.6°

3 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 + 10x + 12y + 35 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(1, −3) lies inside or outside the circle.

Complete the square: $(x + 5)^2 + (y + 6)^2 = 26$
Centre (−5, −6), radius $\sqrt{26}$
Distance^2 from centre to P = $6^2 + 3^2 = 45$
Compare with $r^2 = 26$
Answer: (a) centre (−5, −6), radius $\sqrt{26}$ (b) P is outside the circle

4 Binomial mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X has a binomial distribution with mean 10 and variance 7.5.
(a) Find n and p.
(b) Find P(X = 11) to 3 significant figures.

(a) np = 10 and npq = 7.5
$q = \frac{npq}{np} = \frac{7.5}{10} = 0.75$, so p = 0.25
$n = \frac{10}{0.25} = 40$
(b) $P(X = 11) = C(40, 11)(0.25)^{11}(0.75)^{29} = 0.131$
Answer: (a) n = 40, p = 0.25 (b) 0.131

5 Combined inequalities (All boards)

(a) Solve $4x - 6 < -4$.
(b) Solve $x^2 + 7x + 12 \geq 0$.
(c) Hence find the values of x satisfying both inequalities.

(a) $4x - 6 < -4$ so $x < \frac{1}{2}$
(b) $(x + 4)(x + 3) \geq 0$: critical values −4 and −3
(b) gives $x \leq -4$ or $x \geq -3$
(c) Overlap of the two sets.
Answer: (a) $x < \frac{1}{2}$ (b) $x \leq -4$ or $x \geq -3$ (c) $x \leq -4$ or $-3 \leq x < \frac{1}{2}$

6 Differential equations in context (All boards)

A drink at 92°C is left in a room at 22°C. Its temperature θ °C satisfies $\frac{d\theta}{dt} = -k(\theta - 22)$.

After 15 minutes the temperature is 67°C. Find k, then the time for it to reach 42°C, to 3 s.f.

$\int \frac{1}{\theta - 22}\, d\theta = \int -k\, dt$, so $\ln(\theta - 22) = -kt + C$

t = 0: $\theta - 22 = 70$, so $\theta - 22 = 70e^{-kt}$

t = 15: $45 = 70e^{-15k}$, so $k = \frac{\ln(\frac{14}{9})}{15} = 0.0295$

$20 = 70e^{-kt}$: $t = \frac{\ln(\frac{7}{2})}{k} = 42.5$ minutes

Answer: k = 0.0295, t = 42.5 minutes

7 Range of horizontal force on a rough slope (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg particle is in equilibrium on a rough plane inclined at α, $\tan\alpha = \frac{8}{15}$, μ = 0.2. A horizontal force of P N, in the vertical plane through a line of greatest slope, pushes it towards the plane. Find the least and greatest values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{8}{17}$, $\cos\alpha = \frac{15}{17}$, weight W = 39.2 N. Normal reaction $R = W\cos\alpha + P\sin\alpha$

Least P: about to slide down, friction F = μR up the slope

$P\cos\alpha + \mu(W\cos\alpha + P\sin\alpha) = W\sin\alpha \rightarrow P = W(\sin\alpha - \mu\cos\alpha) \div (\cos\alpha + \mu\sin\alpha) = 11.8$

Greatest P: about to slide up, friction down the slope

$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) \rightarrow P = W(\sin\alpha + \mu\cos\alpha) \div (\cos\alpha - \mu\sin\alpha) = 32.2$

Answer: least 11.8 N, greatest 32.2 N

8 Conditional binomial probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(16, 0.3). Find $P(X = 4 \mid X \geq 2)$ to 3 significant figures.

$P(X = 4 \mid X \geq 2) = \frac{P(X = 4)}{P(X \geq 2)}$ (since X = 4 implies $X \geq 2$)

$P(X = 4) = 0.204$

$P(X \geq 2) = 1 - P(X \leq 1) = 0.974$

Ratio = 0.210

Answer: 0.210

9 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2\theta + 6\cot\theta - 8 = 0$

for $0° < \theta < 360°$. Give answers to 1 d.p.

Use $\text{cosec}^2\theta \equiv 1 + \cot^2\theta$: $4\cot^2\theta + 6\cot\theta - 4 = 0$

$\cot\theta = \frac{1}{2}$ or $\cot\theta = -2$

$\cot\theta = \frac{1}{2}$ means $\tan\theta = 2$: θ = 63.4°, 243.4°

$\cot\theta = -2$ means $\tan\theta = -\frac{1}{2}$: θ = 153.4°, 333.4°

Answer: θ = 63.4°, 153.4°, 243.4°, 333.4°

10 Sum to infinity: two possible ratios (All boards)

The second term of a geometric series is 3 and its sum to infinity is 16.
Find the two possible values of the first term.

$ar = 3$ and $\frac{a}{(1 - r)} = 16$

$a = 16(1 - r)$, so $16(1 - r)\, r = 3$

$16r^2 - 16r + 3 = 0$: $r = \frac{1}{4}$ or $r = \frac{3}{4}$

$a = 16(1 - r) = 12$ or 4

Answer: a = 4 or a = 12