Day 191 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x + 6$ and $g(x) = x^2 - 5$.
Solve $gf(x) = 44$.

$gf(x) = g(x + 6) = (x + 6)^2 - 5$
$(x + 6)^2 - 5 = 44$
$x + 6 = \pm 7$
Answer: $x = -13$ or $x = 1$

2 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The times in seconds of two groups: group X has n = 28, mean 27, sd 9;
group Y has n = 20, mean 63, sd 6.
Find the mean and standard deviation of all 48 values (3 s.f. for the sd).

$\Sigma x = 28 \times 27 + 20 \times 63 = 2016$, mean = 42
Σx^2 (A) $= 28(9^2 + 27^2) = 22680$
Σx^2 (B) $= 20(6^2 + 63^2) = 80100$
Variance $= \frac{102780}{48} - 42^2 = 377.25$, so sd = 19.4
Answer: mean 42, sd 19.4

3 Completing the square (All boards)

$f(x) = 2x^2 - 8x + 5$.
(a) Write f(x) in the form $a(x + p)^2 + q$.
(b) State the minimum value of f(x) and the value of x at which it occurs.
(c) Solve f(x) = 0, giving surds in simplest form.

$f(x) = 2(x - 2)^2 - 3$
(b) minimum value -3 when $x = 2$
(c) $2(x - 2)^2 = 3$, so $(x - 2)^2 = \frac{3}{2}$
$x - 2 = \pm\sqrt{\frac{3}{2}}$
Answer: (a) $2(x - 2)^2 - 3$ (b) minimum -3 at $x = 2$ (c) $x = 2 \pm \frac{\sqrt{6}}{2}$

4 Double and triple angle values (All boards)

θ is acute and $\tan\theta = 6$.
Show that $\cos 2\theta = -\frac{35}{37}$ and find the exact value of $\sin 2\theta$.

$\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1 - 6^2}{1 + 6^2} = -\frac{35}{37}$
$\sin 2\theta = \frac{2\tan\theta}{1 + \tan^2\theta} = \frac{2 \times 6}{1 + 6^2} = \frac{12}{37}$
Answer: $\cos 2\theta = -\frac{35}{37}$, $\sin 2\theta = \frac{12}{37}$

5 Continuous random variable: finding k and E(X) (CIE)

The random variable X has probability density function $f(x) = k(3 - x)$ for $0 \le x \le 3$, and 0 otherwise.
(a) Find k. (b) Find E(X). (c) Find P(X > 1).

(a) $\int_0^3 f(x)\,dx = 1$ gives $\frac{9}{2}k = 1$, so $k = \frac{2}{9}$
(b) $E(X) = \int x\,f(x)\,dx = 1$
(c) $P(X > 1) = \int_1^3 f(x)\,dx = \frac{4}{9}$
Answer: (a) $k = \frac{2}{9}$ (b) 1 (c) $\frac{4}{9}$

6 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 800 kg car tows a 400 kg trailer by a light tow-bar on a horizontal road. Driving force 1150 N; resistances 400 N (car), 150 N (trailer). Find (a) the acceleration and the tow-bar tension. At 10 m s^{-1} the tow-bar breaks. (b) How far does the trailer then travel? Give answers to 3 s.f. if not exact.

Whole system: $1150 - 400 - 150 = 1200\,a$, so $a = 0.5$

Trailer: $T - 150 = 400\,a$, so $T = 150 + 400\times0.5 = 350$

(b) After the break the trailer decelerates at $150 \div 400 = 0.375$ m s^{-2}

$0 = 10^2 - 2\times0.375\times s$, so $s = \frac{400}{3}$

Answer: (a) 0.5 m s^{-2}, 350 N (b) 133 m

7 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix}1\\6\\0\end{pmatrix} + t\begin{pmatrix}1\\-2\\2\end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(5, 2, 3) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathrm{PF} = \begin{pmatrix}t-4\\-2t+4\\2t-3\end{pmatrix}$

$\mathrm{PF}\cdot\mathbf{d} = 0$ gives $9t - 18 = 0$, so $t = 2$

F = (3, 2, 4); $\mathrm{PF} = \begin{pmatrix}-2\\0\\1\end{pmatrix}$, so $|\mathrm{PF}| = \sqrt{5}$

Answer: F(3, 2, 4), distance $\sqrt{5}$

8 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $4x - 3y = 7$ meets the circle $(x-4)^2 + (y-2)^2 = 25$ at the points A and B. Find the exact length of AB.

Centre (4, 2), radius 5. Distance from centre to line:

$d = \frac{|3|}{5} = \frac{3}{5}$

$\text{Half-chord}^2 = 25 - (\frac{3}{5})^2 = \frac{616}{25}$

$\mathrm{AB} = 2\sqrt{\frac{616}{25}}$

Answer: $\frac{20\sqrt{154}}{25}$

9 Velocity, rest and total distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on the x-axis with velocity $v = (3t^2 - 12t + 9)$ m s^{-1} at time t s, $t \geq 0$, and starts at O. Find (a) the times when P is at instantaneous rest, (b) the total distance P travels in the first 6 s.

(a) $v = 3(t-1)(t-3) = 0$, so $t = 1$ or $t = 3$

(b) Displacement $x = \int v\,dt = t^3 - 6t^2 + 9t$

$x(1) = 4$, $x(3) = 0$, $x(6) = 54$

Direction changes at $t = 1$ and $t = 3$, so distance $= |4| + |0 - 4| + |54 - 0|$

Answer: (a) t = 1, t = 3 (b) 62 m

10 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $x^2 + xy + 3y^2 = 99$.

Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{2x + y}{x + 6y}$

Horizontal tangent: $2x + y = 0$, so $y = -2x$

Substitute: $11x^2 = 99$, so $x^2 = 9$ and $x = \pm3$

Points: (3, −6) and (−3, 6)

Answer: (3, −6) and (−3, 6)