Day 190 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Geometric distribution (CIE)

On each independent trial, the probability that a basketball player scores is $\frac{1}{3}$.

X is the number of trials up to and including the first success.

(a) Find P(X = 4) and E(X).

(b) Find the least m with P(X ≤ m) > 0.9.

(a) $P(X = 4) = (\frac{2}{3})^3 \times \frac{1}{3} = \frac{8}{81}$; $E(X) = \frac{1}{p} = 3$

(b) $P(X \leq m) = 1 - (\frac{2}{3})^m > 0.9$

$(\frac{2}{3})^m < 0.1$, so $m > 5.68$

Least integer m = 6

Answer: (a) $\frac{8}{81}$, E(X) = 3 (b) m = 6

2 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 7 m s^{-1} and accelerates uniformly. She travels 85 m in the next 10 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $85 = 7\times10 + \frac{1}{2}\times a\times10^2$, so a = 0.3

(b) $v = u + at = 7 + 0.3\times10 = 10$

(c) For the midpoint, s = 42.5: $v^2 = u^2 + 2as = 7^2 + 2\times0.3\times42.5$

$v^2 = 74.5$

Answer: (a) 0.3 m s^{-2} (b) 10 m s^{-1} (c) 8.63 m s^{-1}

3 Transformations of a turning point (All boards)

The curve y = f(x) has a minimum point at (−2, −6).

Find the coordinates of the corresponding point on:

(a) y = f(x) − 4

(b) y = f(x + 2)

(c) y = −f(x)

(a) y = f(x) − 4: (−2, −10)

(b) y = f(x + 2): (−4, −6)

(c) y = −f(x): (−2, 6)

A stretch or translation moves the point in the stated direction only.

Answer: (a) (−2, −10); (b) (−4, −6); (c) (−2, 6)

4 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

f(x) = 4 arcsin(3x).

State the domain and range of f, and find $f^{-1}(x)$.

arcsin has domain −1 to 1, so $-1 \leq 3x \leq 1$

Range of arcsin is $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, so f(x) runs from $-\frac{4\pi}{2}$ to $\frac{4\pi}{2}$

y = 4 arcsin(3x) gives $\sin\frac{y}{4} = 3x$, so $f^{-1}(x) = \frac{1}{3}\sin\frac{x}{4}$

Answer: domain $-\frac{1}{3} \leq x \leq \frac{1}{3}$, range $-\frac{4\pi}{2} \leq f(x) \leq \frac{4\pi}{2}$, $f^{-1}(x) = \frac{1}{3}\sin\frac{x}{4}$

5 Sigma notation (All boards)

Evaluate $\sum_{r=7}^{31}(3r + 6)$.

Arithmetic series: first term 27, last term 99, 25 terms

$S = \frac{25}{2}(27 + 99) = 1575$

Answer: 1575

6 Equations with cosec x and cot x (All boards)

Solve $2\text{cosec}^2 x + 10\cot x + 6 = 0$
for $0° < x < 360°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $2\cot^2 x + 10\cot x + 8 = 0$
$\cot x = -1$ or $\cot x = -4$
$\cot x = -1$ means $\tan x = -1$: $x = 135.0°, 315.0°$
$\cot x = -4$ means $\tan x = -\frac{1}{4}$: $x = 166.0°, 346.0°$

Answer: x = 135.0°, 166.0°, 315.0°, 346.0°

7 Circle: tangent at a point (AQA, Edexcel, OCR, MEI, Eduqas)

The circle $x^2 + y^2 - 8x + 4y - 5 = 0$ passes through P(0, 1).
(a) Find the equation of the tangent at P, in the form $ax + by = c$.
(b) Find the coordinates of the point where this tangent crosses the x-axis.

Centre $(4, -2)$. Gradient of radius CP $= -\frac{3}{4}$

Gradient of tangent $= \frac{4}{3}$

$y - 1 = \frac{4}{3}(x\ )$: $4x - 3y = -3$

$y = 0$: $x = -\frac{3}{4}$

Answer: (a) $4x - 3y = -3$ (b) $(-\frac{3}{4}, 0)$

8 Quotient rule: stationary points (All boards)

Find the coordinates of the stationary points of

$y = \frac{x}{x^2 + 1}$.

$\frac{dy}{dx} = \frac{(x^2 + 1) \times 1 - x \times 2x}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$

$\frac{dy}{dx} = 0$ when $x^2 = 1$, so $x = \pm 1$

$x = 1$: $y = \frac{1}{2} = \frac{1}{2}$; $x = -1$: $y = -\frac{1}{2}$

Answer: $(-1, -\frac{1}{2})$, $(1, \frac{1}{2})$

9 Regression line from summary data (AQA, Edexcel, OCR, MEI, Eduqas)

n = 10 pairs of (x, y): $\Sigma x = 154$, $\Sigma y = 475$, $\Sigma x^2 = 2942$, $\Sigma xy = 8708$.
(a) Find the equation of the regression line of y on x (3 s.f.).
(b) Estimate y when x = 11.

$S_{xx} = 2942 - \frac{154^2}{10} = 570$, $S_{xy} = 8708 - \frac{154 \times 475}{10} = 1390$

$b = \frac{S_{xy}}{S_{xx}} = 2.44$
$a = \bar{y} - b\,\bar{x} = 47.5 - (2.44)(15.4) = 9.89$
(b) $y = 9.89 + (2.44)(11) = 36.8$
Answer: (a) $y = 9.89 + 2.44x$ (b) 36.8

10 Closest approach of two particles (Edexcel, OCR, MEI)

At t = 0, P and Q have position vectors $(-3\mathbf{i} + 9\mathbf{j})$ m and $(-\mathbf{i} + 3\mathbf{j})$ m and velocities $(-\mathbf{i} - 2\mathbf{j})$ m s^{-1} and $(-3\mathbf{i})$ m s^{-1}. Find (a) the time when they are closest, (b) the least distance between them. Give answers to 3 s.f. if not exact.

$\mathrm{PQ} = \mathrm{r_P} - \mathrm{r_Q} = (-2\mathbf{i} + 6\mathbf{j}) + t(2\mathbf{i} - 2\mathbf{j})$

$d^2 = (2t - 2)^2 + (-2t + 6)^2 = 8t^2 - 32t + 40$

(a) $d(d^2)/dt = 0$: $16t + -32 = 0$, so $t = 2$

(b) At $t = 2$: $\mathrm{PQ} = 2\mathbf{i} + 2\mathbf{j}$, so $d = \sqrt{8}$

Answer: (a) t = 2 s (b) 2.83 m