Day 19 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 19 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{95}{13}, \frac{228}{13})$. At time T the velocity is $(\frac{95}{13}, \frac{228}{13} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{95}{13})^2 + \frac{228}{13}(\frac{228}{13} - 9.8T) = 0$

$19^2 - 9.8T \times \frac{228}{13} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 19 \div (9.8 \times \frac{12}{13}) = \frac{1235}{588}$

(b) $h = \frac{228}{13} \times \frac{1235}{588} - 4.9 \times \frac{1235}{588}^2 = \frac{30685}{2016}$

Answer: (a) T = 2.10 s (b) 15.2 m

2 Geometric distribution (CIE)

On each independent trial, the probability that a bulb is faulty is $\frac{1}{6}$.

X is the number of trials up to and including the first success.
(a) Find P(X = 4) and E(X).
(b) Find the least m with $P(X \leq m) > 0.95$.

(a) $P(X = 4) = (\frac{5}{6})^3 \times \frac{1}{6} = \frac{125}{1296}$; $E(X) = \frac{1}{p} = 6$

(b) $P(X \leq m) = 1 - (\frac{5}{6})^m > 0.95$

$(\frac{5}{6})^m < 0.05$, so $m > 16.4$

Least integer m = 17

Answer: (a) $\frac{125}{1296}$, E(X) = 6 (b) m = 17

3 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 - 4x - 4y - 10 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(2, −2) lies inside or outside the circle.

Complete the square: $(x - 2)^2 + (y - 2)^2 = 18$

Centre (2, 2), radius $3\sqrt{2}$

Distance^2 from centre to $P = 0^2 + (-4)^2 = 16$

Compare with $r^2 = 18$

Answer: (a) centre (2, 2), radius $3\sqrt{2}$ (b) P is inside the circle

4 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean visibility (Dm) in the same month are:
Camborne: mean 24, standard deviation 3.9;
Leeming: mean 6, standard deviation 4.6.
Make two comparisons.

Means: Camborne has the higher mean (24 against 6), so on average the mean visibility is greater at Camborne

Standard deviations: Leeming is larger at Leeming (4.6 against 3.9)

so the readings at Leeming are more variable and those at Camborne are more consistent

Answer: Camborne higher on average; Leeming more variable

5 Chain rule and gradients (All boards)

$y = (\ln x)^2$ for $x > 0$.

Find $\frac{dy}{dx}$ and its exact value at $x = e$.

$\frac{dy}{dx} = 2 \ln x \times \frac{1}{x} = \frac{2 \ln x}{x}$

At $x = e$: $\ln x = 1$, so gradient $= \frac{2}{e}$

Answer: $\frac{dy}{dx} = \frac{2 \ln x}{x}$; gradient $= \frac{2}{e}$

6 Work-energy on a rough slope (CIE)

A 3 kg block is released from rest on a rough plane inclined at α, $\tan \alpha = \frac{5}{12}$, $\mu = 0.2$, and slides 4 m down a line of greatest slope. Use work and energy to find (a) the work done against friction, (b) its speed after 4 m. Take $g = 10$. Give answers to 3 s.f. if not exact.

$R = 3g \cos \alpha = \frac{360}{13}$ N, $F = \mu R = \frac{72}{13}$ N

(a) Work done against friction $= F \times d = \frac{72}{13} \times 4$

(b) Loss of PE $= 3g \times 4 \sin \alpha = \frac{600}{13}$ J

Work-energy: $\frac{1}{2} \times 3 \times v^2 = \frac{600}{13} - \frac{288}{13} = 24$

$v^2 = 16$

Answer: (a) 22.2 J (b) 4 m s^{-1}

7 Line and curve intersection (diagram) (All boards)

The diagram shows the curve $y = x^2 - 10$ and the line $y = -2x - 2$, meeting at A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the midpoint of AB.

$x^2 - 10 = -2x - 2$: $x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$, $x = -4$ or 2

A$(-4, 6)$, B$(2, -6)$

Midpoint: average the coordinates

Answer: (a) A$(-4, 6)$, B$(2, -6)$ (b) $(-1, 0)$

8 Implicit differentiation: products and cubes (All boards)

A curve has equation $2x^2y + y^3 + 2x = -5$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at $(2, -1)$.

Differentiate: $4xy + 2x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} + 2 = 0$

$\frac{dy}{dx} = -\frac{4xy + 2}{2x^2 + 3y^2}$

At $(2, -1)$: $\frac{dy}{dx} = \frac{6}{11}$, so the normal has gradient $-\frac{11}{6}$

Answer: $\frac{dy}{dx} = -\frac{4xy + 2}{2x^2 + 3y^2}$; normal gradient $-\frac{11}{6}$

9 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$3\sin 3\theta + 4\cos\theta = 8$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin 3\theta \approx 3\theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$

$3 \times 3\theta + 4(1 - \frac{1}{2}\theta^2) = 8$

Multiply by 2 and rearrange: $4\theta^2 - 18\theta + 8 = 0$

Divide through by 2: $2\theta^2 - 9\theta + 4 = 0$

$\theta = \frac{9 \pm \sqrt{49}}{4}$, so $\theta = 0.500$

Answer: a = 2, b = −9, c = 4; $\theta \approx 0.500$

10 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A(3, 13) and B(−7, 3).
Its centre lies on the line $y = 2x - 3$.
Find the equation of the circle.

Centre $(h, 2h - 3)$ is equidistant from A and B.

$(h - 3)^2 + (2h - 16)^2 = (h + 7)^2 + (2h - 6)^2$

Solving gives h = 3, so centre (3, 3)

$r^2 = (0)^2 + (10)^2 = 100$

Answer: $(x - 3)^2 + (y - 3)^2 = 100$