Day 189 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show by counter-example that the statement "The product of two irrational numbers is irrational" is false.

Take $\sqrt{2}$ and $\sqrt{2}$, both irrational.
Their product is 2, which is rational.
Answer: $\sqrt{2} \times \sqrt{2} = 2$

2 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$. The point P lies on AB with AP : PB = 2 : 1.

Find the position vector of P and the exact distance OP.

$\text{OP} = \text{OA} + \frac{2}{3}\text{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$\text{OP} = \sqrt{2^2 + (-3)^2} = \sqrt{13} = \sqrt{13}$

Answer: $\begin{pmatrix} \mathbf{2} \\ \mathbf{-3} \end{pmatrix}$**, OP =** $\sqrt{\mathbf{13}}$

3 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 24.5 m s^{-1} from a point h m above horizontal ground, and hits the ground 7 s later. Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: s = −h, u = 24.5, a = −9.8, t = 7
$-h = 24.5\times7 - 4.9\times7^2$, so h = 68.6
(b) At the highest point v = 0: $0 = 24.5^2 - 2\times9.8\times s$
s = 30.625 m above the point of projection
Greatest height above the ground = 68.6 + 30.625
Answer: (a) 68.6 m (b) 99.225 m

4 Double angle equations (All boards)

Solve cos 2x + 5sin x = 3
for $0 \leq x \leq 2\pi$.

$\cos 2x \equiv 1 - 2\sin^2 x$: $2\sin^2 x - 5\sin x + 2 = 0$

Solve the quadratic: $\sin x = \frac{1}{2}$ or $\sin x = 2$

$\sin x = \frac{1}{2}$: $x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin x = 2$ is impossible

Answer: $\mathbf{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

5 Partial fractions: repeated factor (All boards)

Express $\frac{7x^2 + 22x - 8}{(x-2)(x+2)^2}$ in the form $\frac{A}{(x-2)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$, and find A, B and C.

$7x^2 + 22x - 8 \equiv A(x+2)^2 + B(x-2)(x+2) + C(x-2)$

$x = 2$: $64 = A(4)^2$ so A = 4

$x = -2$: $-24 = C(-4)$ so C = 6

Coefficient of x^2: 7 = A + B so B = 3

Answer: A = 4, B = 3, C = 6

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6 Hypothesis test for a normal mean (All boards)

The mean time taken (minutes) is normally distributed with standard deviation 5; it was previously 20.
A random sample of 16 has mean 22.3.
Test, at the 5% level, whether the mean has increased.

H_0: $\mu = 20$, H_1: $\mu > 20$

$\bar{X} \sim N(20, \frac{5^2}{16})$, $z = \frac{22.3 - 20}{\frac{5}{4}} = 1.84$

p-value = 0.0329 < 0.05
Reject H_0: evidence that the mean has increased.
Answer: z = 1.84, p = 0.0329; reject H_0

7 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (20 kg, 2 m) is smoothly hinged at A to a vertical wall, with a 4 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{4}{3}$. Find (a) the tension, (b) the magnitude of the hinge force. Take g = 9.8.

Moments about A: $T \sin\theta \times 2 = 20\,g \times 1 + 4\,g \times 2$
$T \sin\theta = (10 + 4)g = 137.2$, so (a) $T = 172$
Horizontal component of hinge force: $X = T\cos\theta = 103$
Vertical: $Y + T\sin\theta = (20 + 4)g$, so $Y = 20\,g \div 2 = 98$
(b) Force $= \sqrt{(X^2 + Y^2)} = \sqrt{(103^2 + 98^2)}$
Answer: (a) 171.5 N (b) 142.1 N

8 Continuous random variable: variance (CIE)

X has pdf $f(x) = \frac{6}{8}x(2 - x)$ for $0 \le x \le 2$, and 0 otherwise.
Find E(X), Var(X) and the standard deviation of X (3 s.f.).

$E(X) = \int_0^2 x\,f(x)\,dx = 1$

$E(X^2) = \int_0^2 x^2\,f(x)\,dx = \frac{6}{5}$

$\text{Var}(X) = \frac{6}{5} - (1)^2 = \frac{1}{5}$

Standard deviation $= \sqrt{\frac{1}{5}} = 0.447$

Answer: E(X) = 1, Var(X) = $\frac{1}{5}$, sd 0.447

9 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(4 - x)^{-\frac{1}{2}}$ in ascending powers of x, up to and including x^2.
State the range of x for which the expansion is valid.

$(4 - x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}(1 + \frac{-1x}{4})^{-\frac{1}{2}}$

$4^{-\frac{1}{2}} = \frac{1}{2}$

Expand $(1 + y)^{-\frac{1}{2}}$ with $y = \frac{-1x}{4}$

Valid for $|\frac{-1x}{4}| < 1$, i.e. $|x| < 4$

Answer: $\frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2$; $|x| < 4$

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10 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $3x^2 + xy + 3y^2 = 105$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$$\frac{dy}{dx} = -\frac{6x + y}{x + 6y}$$

Horizontal tangent: $6x + y = 0$, so $y = -6x$

Substitute: $105x^2 = 105$, so $x^2 = 1$ and $x = \pm 1$

Points: $(1, -6)$ and $(-1, 6)$

Answer: $(1, -6)$ and $(-1, 6)$