Day 188 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 3rd term of an arithmetic sequence is 5 and the 7th term is 21.
Find the first term and common difference, and the sum of the first 12 terms.

$u_7 - u_3 = 4\ d = 21 - 5$ so $d = 4$

$a + 2d = 5$ so $a = -3$

$S_{12} = \frac{12}{2}(2 \times (-3) + 11 \times 4)$

Answer: $a = -3$, $d = 4$, $S_{12} = 228$

2 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix}-3\\-5\end{pmatrix}$ and $\begin{pmatrix}5\\3\end{pmatrix}$. The point P lies on AB with AP : PB = 3 : 1.

Find the position vector of P and the exact distance OP.

$\overrightarrow{OP} = \overrightarrow{OA} + \frac{3}{4}\overrightarrow{AB} = \begin{pmatrix}-3\\-5\end{pmatrix} + \frac{3}{4}\begin{pmatrix}8\\8\end{pmatrix}$

$= \begin{pmatrix}3\\1\end{pmatrix}$

$OP = \sqrt{3^2 + 1^2} = \sqrt{10} = \sqrt{10}$

Answer: $\begin{pmatrix}3\\1\end{pmatrix}$, OP = $\sqrt{10}$

3 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 8 m s^{-1} and accelerates uniformly. She travels 100 m in the next 10 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $100 = 8\times10 + \frac{1}{2}\times a\times10^2$, so $a = 0.4$

(b) $v = u + at = 8 + 0.4\times10 = 12$

(c) For the midpoint, $s = 50$: $v^2 = u^2 + 2as = 8^2 + 2\times0.4\times50$

$v^2 = 104$

Answer: (a) 0.4 m s^{-2} (b) 12 m s^{-1} (c) 10.2 m s^{-1}

4 Equations quadratic in disguise (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^4 - 17x^2 + 16 = 0$.

Let $u = x^2$: $u^2 - 17u + 16 = 0$

$(u - 16)(u - 1) = 0$ so $u = 16$ or $u = 1$

$x^2 = 16$ gives $x = \pm4$

$x^2 = 1$ gives $x = \pm1$

Answer: $x = \pm1, \pm4$

5 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $\mathbf{F_1} = (-2\mathbf{i} + 5\mathbf{j})$ N, $\mathbf{F_2} = (8\mathbf{i} - 15\mathbf{j})$ N and $\mathbf{F_3} = (a\mathbf{i} + b\mathbf{j})$ N keep a 2 kg particle in equilibrium. Find (a) a and b. $\mathbf{F_2}$ is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = \mathbf{0}$

$\mathbf{i}$: $-2 + 8 + a = 0$, so $a = -6$

$\mathbf{j}$: $5 + (-15) + b = 0$, so $b = 10$

(b) The resultant is now $\mathbf{F_1} + \mathbf{F_3} = -\mathbf{F_2}$, of magnitude $\sqrt{(8^2 + -15^2)} = 17$ N

$a = 17 \div 2$

Answer: (a) $a = -6$, $b = 10$ (b) 8.5 m s^{-2}

6 Finding an unknown standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is normally distributed with mean 71 and standard deviation σ.
Given that P(X > 79) = 0.1587, find σ to 3 significant figures.

P(Z > z) = 0.1587, so z = 1.00

$\frac{79 - 71}{\sigma} = 1.00$

$\sigma = \frac{8}{1.00} = 8.00$

Answer: σ = 8.00

7 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $2x^2 - 2xy - 2y^2 + x - 4y = 4$.
Show that P(−2, 1) lies on the curve and find the equation of the tangent at P in the form ax + by + c = 0 with integer a, b, c.

Differentiate (product rule on the xy term): $(4x - 2y + 1) + (-2x - 4y - 4)\frac{dy}{dx} = 0$

At P(−2, 1): $-9 + (-4)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{9}{4}$

Tangent: $y - 1 = (-\frac{9}{4})(x + 2)$, i.e. 9x + 4y + 14 = 0

Answer: 9x + 4y + 14 = 0

8 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the score (marks), is normally distributed with mean 62 and standard deviation 12.
Find (a) P(X > 53.6) (b) P(53.6 < X < 63.2), each to 3 s.f.

(a) $z = \frac{53.6 - 62}{12} = -0.700$, P(Z > −0.700) = 0.758

(b) $z_1 = -0.700$, $z_2 = \frac{63.2 - 62}{12} = 0.100$

P(53.6 < X < 63.2) = Φ(0.100) − Φ(−0.700) = 0.298

Answer: (a) 0.758 (b) 0.298

9 Cubic inequalities (All boards)

Given that (x + 4) is a factor of $f(x) = -x^3 + x^2 + 14x - 24$, solve f(x) ≥ 0.

Factorise: f(x) = −(x + 4)(x − 2)(x − 3)
Roots at x = −4, 2, 3
Sketch the cubic (negative x^3 coefficient: positive, negative, positive, negative)
Read off where f(x) ≥ 0.

Answer: x ≤ −4 or 2 ≤ x ≤ 3

10 Differential equations in context (All boards)

Water drains from a tank so that the depth h metres falls at a rate proportional to $\sqrt{h}$. Initially h = 49, and after 3 hours h = 4.
Form and solve a differential equation to find how long, in total, the tank takes to empty.

$\frac{dh}{dt} = -k\sqrt{h}$: $\int h^{-\frac{1}{2}}\,dh = \int -k\,dt$, so $2\sqrt{h} = -kt + C$

t = 0: C = 2 × 7 = 14, so $2\sqrt{h} = 14 - kt$

t = 3, h = 4: 4 = 14 − 3k, so $k = \frac{10}{3}$

Empty when h = 0: kt = 14, so $t = \frac{14}{\frac{10}{3}} = \frac{21}{5}$ hours

Answer: $\frac{21}{5}$ hours