Day 187 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 5th term of an arithmetic sequence is 20 and the 11th term is 62.
Find the first term and common difference, and the sum of the first 25 terms.

$u_{11} - u_5 = 6\,d = 62 - 20$ so $d = 7$

$a + 4\,d = 20$ so $a = -8$

$S_{25} = \frac{25}{2}(2 \times (-8) + 24 \times 7)$

Answer: $a = -8$, $d = 7$, $S_{25} = 1900$

2 Finding an unknown standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is normally distributed with mean 45 and standard deviation σ.
Given that $P(X > 48) = 0.0668$, find σ to 3 significant figures.

$P(Z > z) = 0.0668$, so $z = 1.50$

$\frac{48 - 45}{\sigma} = 1.50$

$\sigma = \frac{3}{1.50} = 2.00$

Answer: $\sigma = 2.00$

3 Index equations solved with logarithms (All boards)

Solve $3^{4x} = 7^{2x+2}$.
Give your answer to 3 significant figures.

Take logs: $4x \ln 3 = (2x + 2) \ln 7$

Collect x terms: $x(4 \ln 3 - 2 \ln 7) = 2 \ln 7$

$x = \frac{2 \ln 7}{4 \ln 3 - 2 \ln 7}$

$x = 7.74$

Answer: $x = 7.74$

4 Conservation of momentum (CIE)

Particles A (5 kg) and B (5 kg) move towards each other on a smooth horizontal line at 6 m s^{-1} and 5 m s^{-1}. They collide directly, and A then moves in the opposite direction at 2 m s^{-1}. Find (a) the speed of B after the collision, (b) the magnitude of the impulse of A on B.

Take the initial direction of A as positive. Conservation of momentum:

$5 \times 6 + 5 \times (-5) = 5 \times (-2) + 5\,v$

$5 = -10 + 5\,v$, so $v = 3$

(b) Impulse on B = change in momentum of B $= 5(3 - (-5))$

$= 5 \times 8$

Answer: (a) 3 m s^{-1} (b) 40 N s

5 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix}6\\3\end{pmatrix}$ and $\begin{pmatrix}-3\\6\end{pmatrix}$. The point P lies on AB with AP : PB = 1 : 2.

Find the position vector of P and the exact distance OP.

$\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB} = \begin{pmatrix}6\\3\end{pmatrix} + \frac{1}{3}\begin{pmatrix}-9\\3\end{pmatrix}$

$= \begin{pmatrix}3\\4\end{pmatrix}$

$OP = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

Answer: $\begin{pmatrix}3\\4\end{pmatrix}$, OP = 5

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6 Sum to infinity: two possible ratios (All boards)

The second term of a geometric series is 5 and its sum to infinity is 36.
Find the two possible values of the first term.

$ar = 5$ and $\frac{a}{(1-r)} = 36$

$a = 36(1-r)$, so $36(1-r)\,r = 5$

$36r^2 - 36r + 5 = 0$: $r = \frac{1}{6}$ or $r = \frac{5}{6}$

$a = 36(1-r) = 30$ or 6

Answer: a = 6 or a = 30

7 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Eggs are defective with probability 0.25, independently. A tray of 10 eggs is rejected if it has at least 2 defective.
From 8 such trays, find the probability that exactly 4 are rejected (3 s.f.).

$Y \sim B(10, 0.25)$. $P(\text{rejected}) = P(Y \geq 2) = 1 - 0.244 = 0.756$

$W \sim B(8, 0.756)$ is the number rejected

$P(W = 4) = C(8, 4)(0.756)^4(1-0.756)^4 = 0.0811$

Answer: 0.0811

8 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$5\sin\theta + 4\cos\theta = 6.5$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$

$5 \times \theta + 4(1 - \frac{1}{2}\theta^2) = 6.5$

Multiply by 2 and rearrange: $4\theta^2 - 10\theta + 5 = 0$

$\theta = \frac{10 \pm \sqrt{20}}{8}$, so $\theta = 0.691$

Answer: a = 4, b = −10, c = 5; $\theta \approx 0.691$

9 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1-3x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.
(b) Use $x = 0.01$ to estimate $\sqrt{97}$ to 4 decimal places.

$(1+y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, $y = -3x$

(a) $1 - \frac{3}{2}x - \frac{9}{8}x^2$

$x = \frac{1}{100}$: $97 = 10^2(1 - \frac{3}{100})$ so $\sqrt{97} = 10 \times (1 - \frac{3}{100})^{\frac{1}{2}}$

$\approx 10 \times (1 - \frac{3}{200} - \frac{9}{80000}) = 9.848875$

Answer: (a) $1 - \frac{3}{2}x - \frac{9}{8}x^2$ (b) 9.8489

10 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 10 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take g = 9.8.

$\sin\alpha = 0.6$, $\cos\alpha = 0.8$, $\sin\beta = \frac{5}{13}$, $\cos\beta = \frac{12}{13}$

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_2 = \frac{13}{15} T_1$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 10\,g = 98$

$T_1(0.6 + \frac{1}{3}) = 98$

$T_1 = 105$, $T_2 = 91.0$

Answer: $T_1 = 105$ N, $T_2 = 91$ N