Day 186 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show by counter-example that the statement "If $x^2 > 49$ then $x > 7$" is false.

Take $x = -8$:

$x^2 = 64 > 49$, but $x = -8$ is not greater than 7.

Answer: $x = -8$

2 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For n = 10 pairs of (x, y): $\Sigma x = 172$, $\Sigma y = 712$,

$\Sigma x^2 = 3630$, $\Sigma y^2 = 56922$, $\Sigma xy = 14225$.

Calculate the product moment correlation coefficient, and describe the correlation.

$$S_{xx} = 3630 - \frac{172^2}{10} = 672$$

$$S_{yy} = 56922 - \frac{712^2}{10} = 6230$$

$$S_{xy} = 14225 - \frac{172 \times 712}{10} = 1980$$

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = 0.967$$

This is strong positive correlation

Answer: r = 0.967, positive correlation

3 Differentiating a^x (All boards)

$y = 2 \times 3^{3x}$.

Find the exact gradient of the curve at x = 1 and the equation of the tangent there.

$$\frac{dy}{dx} = 2 \times 3 \ln 3 \times 3^{3x} = 6 \ln 3 \times 3^{3x}$$

At x = 1: y = 54 and gradient = $162 \ln 3$

Tangent: $y = (162 \ln 3)x + 54 - 162 \ln 3$

Answer: gradient $162 \ln 3$; $y = (162 \ln 3)x + 54 - 162 \ln 3$

4 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $4x + 3y = k$ is a tangent to the circle

$(x - 4)^2 + (y - 4)^2 = 9$.

Find the two possible values of k.

Centre (4, 4), radius 3

Distance from centre to the line = radius:

$$\frac{|28 - k|}{5} = 3$$

$|28 - k| = 15$

Answer: k = 13 or k = 43

5 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small x (in radians), use the small angle approximations to find the value that

$$\frac{\sin 3x \tan 4x}{1 - \cos 3x}$$

approaches as $x \to 0$.

$$\sin 3x \approx 3x, \tan 4x \approx 4x, 1 - \cos 3x \approx \frac{1}{2}(3x)^2$$

$$\text{Expression} \approx \frac{3x \times 4x}{\frac{9}{2}x^2} = \frac{12x^2}{\frac{9}{2}x^2} = \frac{8}{3}$$

Answer: $\frac{8}{3}$

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6 Non-uniform rod and equal tensions (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB (length 2 m, mass 8 kg) has its centre of mass at G, AG = 0.75 m. It hangs horizontally from vertical strings at A and B. A particle of mass m kg is fixed at P, AP = 1.5 m, making the tensions equal. Find (a) m, (b) the tension. Take g = 9.8.

Equal tensions T: $2T = (8 + m)g$, so $T = \frac{1}{2}(8 + m)g$
Moments about A: $T \times 2 = 8g \times 0.75 + mg \times 1.5$
$\frac{1}{2}(8 + m) \times 2 = 6 + 1.5m \rightarrow 8 + 1m = 6 + 1.5m$
$m(1.5 - 1) = 8 - 6$, so $m = 4$
(b) $T = \frac{1}{2}(8 + 4) \times 9.8$
Answer: (a) m = 4 (b) 58.8 N

7 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(36, 0.5)$.
Find the smallest value of c such that $P(X \geq c) < 0.1$.

$P(X \geq 23) = 1 - P(X \leq 22) = 0.0662$
$P(X \geq 22) = 1 - P(X \leq 21) = 0.121$
$0.0662 < 0.1$ but $0.121 > 0.1$
Answer: c = 23

8 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{6} - 2) = 4 - \sqrt{6}$, writing x in the form $p + q\sqrt{6}$.

$$x = \frac{4 - \sqrt{6}}{\sqrt{6} - 2}$$

Multiply top and bottom by $\sqrt{6} + 2$.
Numerator: $(4 - \sqrt{6})(\sqrt{6} + 2) = 2 + 2\sqrt{6}$
Denominator: $6 - 4 = 2$
Answer: $x = 1 + \sqrt{6}$

9 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (20 kg, 4 m) is smoothly hinged at A to a vertical wall, with a 2 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{5}{12}$. Find (a) the tension, (b) the magnitude of the hinge force. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about A: $T \sin\theta \times 4 = 20g \times 2 + 2g \times 4$
$T \sin\theta = (10 + 2)g = 117.6$, so (a) $T = 306$
Horizontal component of hinge force: $X = T\cos\theta = 282$
Vertical: $Y + T\sin\theta = (20 + 2)g$, so $Y = 20g \div 2 = 98$
(b) Force $= \sqrt{(X^2 + Y^2)} = \sqrt{(282^2 + 98^2)}$
Answer: (a) 306 N (b) 299 N

10 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ 2 \end{pmatrix}$ and $\begin{pmatrix} k \\ 1 \\ -1 \end{pmatrix}$ are perpendicular.
Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 + k - 2 = 0$
$(k + 2)(k - 1) = 0$
$k = -2$ or $k = 1$. For $k = 1$ the first vector is $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, with length $\sqrt{6} = \sqrt{6}$
Answer: $k = -2$ or 1; length $\sqrt{6}$