Day 185 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (−1, −2, 0), (−1, 0, 1) and (3, 4, 1).
Find angle ABC, in degrees to 1 d.p.

$\text{BA} = \begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix}$, $\text{BC} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}$

$\text{BA} \cdot \text{BC} = -8$

$|\text{BA}| = \sqrt{5}$, $|\text{BC}| = \sqrt{32}$

$\cos \text{ABC} = \frac{-8}{\sqrt{5} \times \sqrt{32}} = -0.6325$, so angle ABC = 129.2°

Answer: 129.2°

2 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily total rainfall (mm) in the same month are:
Camborne: mean 28, standard deviation 1.5;
Leeming: mean 6, standard deviation 3.1.
Make two comparisons.

Means: Camborne has the higher mean (28 against 6), so on average the total rainfall is greater at Camborne
Standard deviations: Leeming is larger at Leeming (3.1 against 1.5)
so the readings at Leeming are more variable and those at Camborne are more consistent

Answer: Camborne higher on average; Leeming more variable

3 Modulus equations (All boards)

Solve $|3x - 3| = |2x - 7|$.

Either $3x - 3 = 2x - 7$ or $3x - 3 = -(2x - 7)$
First: $x = -4$
Second: $5x = 10$

Answer: $x = -4$ or $x = 2$

4 Arrangements with restrictions (OCR, MEI, Eduqas, CIE)

How many arrangements of the letters of the word CRAYONS have all the vowels next to each other?

Treat the 2 vowels as one block: 6 items to arrange in 6! ways
The vowels can be ordered within the block in 2! ways
Total = 6! × 2! = 1440

Answer: 1440

5 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 15 kg is pulled at constant speed along rough horizontal ground, $\mu = 0.3$, by a force of P N at angle θ above the horizontal, $\tan \theta = \frac{5}{12}$. Find P and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

Constant speed, so the resultant force is zero. $\sin \theta = \frac{5}{13}$, $\cos \theta = \frac{12}{13}$

Vertical: $R = 15\,g - P \sin \theta$
Horizontal: $P \cos \theta = \mu R = 0.3(15\,g - P \sin \theta)$
$P(\frac{12}{13} + \frac{3}{26}) = 0.3 \times 15\,g$
$P = 42.5$, $R = 131$

Answer: P = 42.5 N, R = 131 N

6 Proving trigonometric identities (All boards)

Prove the identity

$\frac{1}{1-\sin x}+\frac{1}{1+\sin x} \equiv 2\sec^2 x$

LHS $= \frac{(1+\sin x)+(1-\sin x)}{(1-\sin x)(1+\sin x)} = \frac{2}{1-\sin^2 x}$

$= \frac{2}{\cos^2 x} = 2\sec^2 x$ = RHS

Answer: Proved

7 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $-3x + 4y = k$ is a tangent to the circle

$(x-3)^2 + (y-2)^2 = 9$.

Find the two possible values of k.

Centre (3, 2), radius 3

Distance from centre to the line = radius:

$\frac{|-1-k|}{5} = 3$

$|-1-k| = 15$

Answer: $k = -16$ or $k = 14$

8 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations $x = 2\sin t$, $y = 5\cos 2t$.

Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where $t = \frac{\pi}{3}$, in the form $y = mx + c$.

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = -\frac{10\sin 2t}{2\cos t}$

$t = \frac{\pi}{3}$: gradient $m = -5\sqrt{3}$, point $(\sqrt{3}, -\frac{5}{2})$

$c = y - mx = \frac{25}{2}$

Answer: $\frac{dy}{dx} = -\frac{10\sin 2t}{2\cos t}$; $y = -5\sqrt{3}x + \frac{25}{2}$

9 Particles on two smooth inclined planes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest. Find (a) the magnitude of the acceleration and which particle moves down, (b) the tension. Take g = 9.8. Give answers to 3 s.f. if not exact.

A: weight component along plane = 9 g sin 60°; B: 2 g sin 45°

A moves down, B up (9 sin 60° > 2 sin 45°): F = ma for the system

9 g sin 60° − 2 g sin 45° = (9 + 2)a

(a) a = 5.68

(b) For A: 9 g sin 60° − T = 9 a, so T = 25.2

Answer: (a) 5.68 m s^{-2}, A moves down (b) 25.2 N

10 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{3}-1) = 1 - 4\sqrt{3}$, writing x in the form $p + q\sqrt{3}$.

$x = \frac{1-4\sqrt{3}}{\sqrt{3}-1}$

Multiply top and bottom by $\sqrt{3} + 1$.

Numerator: $(1-4\sqrt{3})(\sqrt{3}+1) = -11 - 3\sqrt{3}$

Denominator: $3 - 1 = 2$

Answer: $x = -\frac{11}{2} - \frac{3}{2}\sqrt{3}$