Day 184 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean temperature (°C) in the same month are:
Camborne: mean 15.5, standard deviation 5.8;
Leeming: mean 17.5, standard deviation 1.1.
Make two comparisons.

Means: Leeming has the higher mean (17.5 against 15.5), so on average the mean temperature is greater at Leeming
Standard deviations: Camborne is larger at Camborne (5.8 against 1.1)
so the readings at Camborne are more variable and those at Leeming are more consistent
Answer: Leeming higher on average; Camborne more variable

2 Conservation of momentum (CIE)

Particles A (7 kg) and B (5 kg) move towards each other on a smooth horizontal line at 9 m s^{-1} and 6 m s^{-1}. They collide and coalesce. Find (a) the speed of the combined particle, (b) the kinetic energy lost in the collision. Give answers to 3 s.f. if not exact.

Conservation of momentum (A's direction positive):
$7\times9 - 5\times6 = 12v$, so $v = 2.75$
KE before = $\frac{1}{2}\times7\times9^2 + \frac{1}{2}\times5\times6^2 = 373.5$
KE after = $\frac{1}{2}\times12\times2.75^2 = 45.375$
Loss = $373.5 - 45.375$
Answer: (a) 2.75 m s^{-1} (b) 328 J

3 Index equations solved with logarithms (All boards)

Solve $3^{3x-2} = 7^{2x+2}$.
Give your answer to 3 significant figures.

Take logs: $(3x - 2)\ln 3 = (2x + 2)\ln 7$
Collect x terms: $x(3\ln 3 - 2\ln 7) = 2\ln 7 + 2\ln 3$
$x = \frac{2\ln 7 + 2\ln 3}{3\ln 3 - 2\ln 7}$
$x = -10.2$
Answer: x = −10.2

4 Line meets curve: length of chord (All boards)

The line $y = -2x + 4$ meets the curve $y = x^2 - 5x + 6$ at A and B.
(a) Find the coordinates of A and B.
(b) Find the exact length AB.

$x^2 - 5x + 6 = -2x + 4$ gives $x^2 - 3x + 2 = 0$
$(x - 1)(x - 2) = 0$: x = 1 or 2
A(1, 2), B(2, 0)
$AB^2 = 1^2 + (-2)^2 = 5$
Answer: A(1, 2), B(2, 0); AB = $\sqrt{5}$

5 Independent and mutually exclusive events (All boards)

P(A) = 0.1 and P(A ∪ B) = 0.73.
(a) Given that A and B are independent, find P(B).
(b) Given instead that A and B are mutually exclusive, find P(B).
(c) In the case in (a), find P(A′ ∩ B).

(a) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, so $0.73 = 0.1 + P(B)(1 - 0.1)$
$P(B) = \frac{0.73 - 0.1}{0.9} = 0.7$
(b) $P(A \cap B) = 0$, so $P(B) = 0.73 - 0.1 = 0.63$
(c) $P(A' \cap B) = P(A')P(B) = 0.9 \times 0.7 = 0.63$
Answer: (a) 0.7 (b) 0.63 (c) 0.63

6 Ships on collision courses (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

At 12:00 ship P has position $(14\mathbf{i} - 8\mathbf{j})$ km and velocity $(-3\mathbf{i} + \mathbf{j})$ km h^{-1}; ship Q has position $(-2\mathbf{i} - 20\mathbf{j})$ km and velocity $(\mathbf{i} + 4\mathbf{j})$ km h^{-1}. Show that the ships collide and find the time and position vector of the collision.

At time t hours: $\mathbf{r}_P = (14\mathbf{i} - 8\mathbf{j}) + t(-3\mathbf{i} + \mathbf{j})$, $\mathbf{r}_Q = (-2\mathbf{i} - 20\mathbf{j}) + t(\mathbf{i} + 4\mathbf{j})$

Equal when: $\mathbf{i}$: $14 + -3t = -2 + 1t$, so $t = 4$

$\mathbf{j}$: $-8 + 4\times1 = -4$

Check with Q: $\mathbf{i}$: $-2 + 4\times1 = 2$, $\mathbf{j}$: $-20 + 4\times4 = -4$

Both components agree at $t = 4$, so they collide at $(2\mathbf{i} - 4\mathbf{j})$ km

Answer: t = 4 hours (at 16:00); r = (2i − 4j) km

7 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against x for $y = A\,b^x$.
Find the exact values of A and b.

ln y = ln A + x ln b, a straight line with gradient ln b and intercept ln A

Intercept ln A = 3, so $A = e^3$

Gradient $\ln b = \frac{4-3}{2-0} = \frac{1}{2}$, so $b = e^{\frac{1}{2}}$

Answer: $A = e^3$, $b = e^{\frac{1}{2}}$

8 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{4x-3}{(1-2x)(1-x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1-2x)} + \frac{B}{(1-x)}$: $A = -2$, $B = -1$

(b) $-2(1-2x)^{-1} = -2(1 + 2x + 4x^2 \ldots)$

Add the series for both fractions.

(c) Need $|2x| < 1$ and $|x| < 1$

Answer: (a) $\frac{-2}{(1-2x)} + \frac{-1}{(1-x)}$ (b) $-3 - 5x - 9x^2$ (c) $|x| < \frac{1}{2}$

9 Binomial expansion: rational powers (All boards)

Expand $(1 + x)^{-1}$ in ascending powers of x up to and including the term in x^2.
State the range of values of x for which the expansion is valid.

$(1+y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \ldots$ with $n = -1$, $y = x$

Coefficients of 1, x, x^2: 1, −1, 1

Valid when $|x| < 1$, i.e. $|x| < 1$

Answer: $1 - x + x^2$; $|x| < 1$

10 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

The volume of a cube of edge x cm is increasing at 48 cm^{3}s^{-1}.
Find the rate of increase of its total surface area when x = 3.

$V = x^3$, $\frac{dV}{dx} = 3x^2$, so $\frac{dx}{dt} = \frac{48}{3x^2} = \frac{16}{9}$ at $x = 3$

$S = 6x^2$, $\frac{dS}{dx} = 12x$, so $\frac{dS}{dt} = 12 \times 3 \times \frac{16}{9} = 64$

Answer: 64 cm^2/s