Day 183 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(−6, 11) and B(4, −13) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(4, 11) lies on the circle.

Centre = midpoint of AB = (−1, −1)
$r^2 = (-5)^2 + (12)^2 = 169$
$(x + 1)^2 + (y + 1)^2 = 169$: $x^2 + y^2 + 2x + 2y - 167 = 0$
(b) $CM^2 = (5)^2 + (12)^2 = 169 = r^2$
Answer: (a) $x^2 + y^2 + 2x + 2y - 167 = 0$

2 Arc length and sector area (All boards)

A sector of a circle of radius 15 cm has perimeter 36 cm.
Find (a) the angle of the sector in radians, (b) the area of the sector.

Perimeter = $2r + r\theta$: $36 = 30 + 15\theta$
$\theta = \frac{6}{15} = 0.4$
Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 15^2 \times 0.4 = 45$
Answer: (a) 0.4 rad (b) 45 cm^2

3 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 24 pairs of values of time spent on social media and exam score, r = 0.438.
The critical value for a two-tailed test at the 1% level is 0.5151.
Test, at this level, whether there is any correlation between the two variables.

$H_0: \rho = 0$, $H_1: \rho \neq 0$
Critical value 0.5151
$|r| = 0.438 < 0.5151$
This is not in the critical region, so do not reject H_0.
There is insufficient evidence of correlation.
Answer: Do not reject H_0: insufficient evidence

4 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 300 kg carries a person of mass 40 kg. The cable tension is 3536 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 340 kg), taking upwards as positive: $T - 340\,g = 340\,a$
$3536 - 3332 = 340\,a$, so $a = 0.6$
(a) a is positive, so the acceleration is 0.6 m s^{-2} upwards
(b) Person: $R - 40\,g = 40\,a$, so $R = 40(9.8 + 0.6)$
Answer: (a) 0.6 m s^{-2} upwards (b) 416 N

5 Selections with combinations (OCR, MEI, Eduqas, CIE)

A group of 7 boys and 7 girls. A team of 3 is chosen at random.
Find the probability that the team contains at least one of the girls.

Total teams = C(14, 3) = 364
Teams of all boys = C(7, 3) = 35
P(at least one girl) = $1 - \frac{35}{364}$
$= \frac{47}{52}$
Answer: $\frac{47}{52}$

6 Newton's second law with a time-varying force (AQA, Edexcel, OCR, MEI)

A 6 kg particle moves on a smooth horizontal line. At time t s a single force of $(30 - 6t)$ N acts along the line. When t = 0 its speed is 1 m s^{-1}. Find (a) its speed when t = 7, (b) its greatest speed.

F = ma: $a = (-6t + 30) \div 6$

$v = \int a\,dt = 5t - 0.5t^2 + c$; v = 1 when t = 0

(a) v(7) = 11.5

(b) Greatest speed when a = 0: t = 5

v(5) = 13.5

Answer: (a) 11.5 m s^{-1} (b) 13.5 m s^{-1}

7 Implicit differentiation: products and cubes (All boards)

A curve has equation $3x^2y - y^3 + 4x = -14$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at (1, 3).

Differentiate: $6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} + 4 = 0$

$\frac{dy}{dx} = -\frac{6xy + 4}{3x^2 - 3y^2}$

At (1, 3): $\frac{dy}{dx} = \frac{11}{12}$, so the normal has gradient $-\frac{12}{11}$

Answer: $\frac{dy}{dx} = -\frac{6xy + 4}{3x^2 - 3y^2}$; normal gradient $-\frac{12}{11}$

8 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1 - 2x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.

(b) Use x = 0.02 to estimate $\sqrt{24}$ to 4 decimal places.

$(1 + y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, $y = -2x$

(a) $1 - x - \frac{1}{2}x^2$

$x = \frac{1}{50}$: $24 = 5^2(1 - \frac{1}{25})$ so $\sqrt{24} = 5 \times (1 - \frac{1}{25})^{\frac{1}{2}}$

$\approx 5 \times (1 - \frac{1}{50} - \frac{1}{5000}) = 4.899000$

Answer: (a) $1 - x - \frac{1}{2}x^2$ (b) 4.8990

9 Asymptotes after transformation (All boards)

The curve y = f(x), where $f(x) = \frac{1}{x - 1} - 1$, is translated by $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of y = f(x): x = 1, y = −1

Translate by $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$: x = 4, y = −4

New curve: $y = \frac{1}{x - 4} - 4$

y = 0: $\frac{1}{x - 4} = 4$, so $x - 4 = \frac{1}{4}$

Answer: x = 4, y = −4; crosses x-axis at $(\frac{17}{4}, 0)$

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10 Proving trigonometric identities (All boards)

Prove the identity

$\sec^2 x + \operatorname{cosec}^2 x \equiv 4\operatorname{cosec}^2 2x$

$$\text{LHS} = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} = \frac{1}{\sin^2 x \cos^2 x}$$

$$= \frac{4}{(2\sin x \cos x)^2} = \frac{4}{\sin^2 2x} = \text{RHS}$$

Answer: Proved