Day 182 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 39.2 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{4}{3}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8.

Initial velocity (23.52, 31.36). At time T the velocity is (23.52, 31.36 − 9.8T).
Perpendicular: scalar product = 0: $(23.52)^2 + 31.36(31.36 - 9.8T) = 0$
$39.2^2 - 9.8T \times 31.36 = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)
(a) $T = 39.2 \div (9.8 \times 0.8) = 5$
(b) $h = 31.36 \times 5 - 4.9 \times 5^2 = 34.3$
Answer: (a) T = 5 s (b) 34.3 m

2 Transformations of a turning point (All boards)

The curve y = f(x) has a maximum point at (−2, 3).
Find the coordinates of the corresponding point on:
(a) y = f(−x)
(b) y = f(x) + 5
(c) y = −f(x)

(a) y = f(−x): (2, 3)
(b) y = f(x) + 5: (−2, 8)
(c) y = −f(x): (−2, −3)
A stretch or translation moves the point in the stated direction only.
Answer: (a) (2, 3); (b) (−2, 8); (c) (−2, −3)

3 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 5 m s^{-1} and accelerates uniformly. She travels 110 m in the next 10 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $110 = 5 \times 10 + \frac{1}{2} \times a \times 10^2$, so a = 1.2
(b) $v = u + at = 5 + 1.2 \times 10 = 17$
(c) For the midpoint, s = 55: $v^2 = u^2 + 2as = 5^2 + 2 \times 1.2 \times 55$
$v^2 = 157$
Answer: (a) 1.2 m s^{-2} (b) 17 m s^{-1} (c) 12.5 m s^{-1}

4 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

The volume of a cube of edge x cm is increasing at 27 $cm^3 s^{-1}$.
Find the rate of increase of its total surface area when x = 5.

$V = x^3$, $\frac{dV}{dx} = 3x^2$, so $\frac{dx}{dt} = \frac{27}{3x^2} = \frac{9}{25}$ at x = 5

$S = 6x^2$, $\frac{dS}{dx} = 12x$, so $\frac{dS}{dt} = 12 \times 5 \times \frac{9}{25} = \frac{108}{5}$

Answer: $\frac{108}{5}$ cm^2/s

5 Equation after a sequence of transformations (All boards)

The curve with equation $y = 2x^2 + 6x + 1$ is transformed by a stretch, scale factor 2, parallel to the y-axis, followed by a stretch, scale factor 3, parallel to the x-axis.
Find the equation of the image curve.

Stretch parallel to y: multiply by 2: $y = 4x^2 + 12x + 2$
Stretch parallel to x: replace x by $\frac{x}{3}$: $y = \frac{4}{9}x^2 + 4x + 2$

Answer: $y = \frac{4}{9}x^2 + 4x + 2$

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6 Area and perimeter of a segment (exact) (All boards)

The circle has centre O and radius 20 cm. Angle AOB = $\frac{2\pi}{3}$.

Find the exact area and the exact perimeter of the shaded segment.

Sector area = $\frac{1}{2}r^2\theta = \frac{400\pi}{3}$; triangle OAB = $\frac{1}{2}r^2\sin\theta = 100\sqrt{3}$

Segment area = sector − triangle = $\frac{400\pi - 300\sqrt{3}}{3}$

Chord AB = $2r\sin\frac{\theta}{2} = 20\sqrt{3}$; arc = $r\theta = \frac{40\pi}{3}$

Perimeter = arc + chord = $\frac{60\sqrt{3} + 40\pi}{3}$

Answer: area = $\frac{400\pi - 300\sqrt{3}}{3}$ cm^2, perimeter = $\frac{60\sqrt{3} + 40\pi}{3}$ cm

7 Finding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(\mu, \sigma^2)$, with $P(X < 46) = 0.1587$ and $P(X > 51) = 0.0668$.

Find μ and σ.

z-values: 46: $z = -1.00$; 51: $z = 1.50$

$46 = \mu - 1.00\,\sigma$ and $51 = \mu + 1.50\,\sigma$

Subtract: $5 = 2.50\,\sigma$, so $\sigma = 2.00$

$\mu = 46 + 1.00 \times 2.00 = 48.0$

Answer: $\mu = 48$, $\sigma = 2$

8 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{7} - 3) = -5 + 4\sqrt{7}$, writing x in the form $p + q\sqrt{7}$.

$x = \frac{-5 + 4\sqrt{7}}{\sqrt{7} - 3}$

Multiply top and bottom by $\sqrt{7} + 3$.

Numerator: $(-5 + 4\sqrt{7})(\sqrt{7} + 3) = 13 + 7\sqrt{7}$

Denominator: $7 - 9 = -2$

Answer: $x = -\frac{13}{2} - \frac{7}{2}\sqrt{7}$

9 Hypothesis test for a normal mean (All boards)

The mean fill volume of bottles (ml) is normally distributed with standard deviation 8; it was previously 85.

A random sample of 64 has mean 82.5.

Test, at the 5% level, whether the mean has changed.

$H_0: \mu = 85$, $H_1: \mu \neq 85$

$\bar{X} \sim N(85, \frac{8^2}{64})$, $z = \frac{82.5 - 85}{\frac{8}{8}} = -2.50$

p-value = 0.0124 (two-tailed) < 0.05

Reject H_0: evidence that the mean has changed.

Answer: $z = -2.50$, $p = 0.0124$; reject H_0

10 Arc length and sector area (All boards)

A sector of a circle has arc length 8 cm and area 16 cm^2.

Find the radius of the circle and the angle of the sector, in radians.

Area = $\frac{1}{2}rs$: $16 = \frac{1}{2} \times r \times 8$

$r = \frac{2 \times 16}{8} = 4$ cm

$\theta = \frac{s}{r} = \frac{8}{4} = 2$ rad

Answer: $r = 4$ cm, $\theta = 2$ rad