Day 181 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 - 3$ and $g(x) = 3x + 3$.
Solve $fg(x) = gf(x)$.

$fg(x) = (3x + 3)^2 - 3 = 9x^2 + 18x + 6$
$gf(x) = 3(x^2 - 3) + 3 = 3x^2 - 6$
$9x^2 + 18x + 6 = 3x^2 - 6$ so $6x^2 + 18x + 12 = 0$
Answer: $x = -2$ or $x = -1$

2 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

$|a| = 6$, $|b| = 4$ and the angle between a and b is 150°.
Find the exact values of $a \cdot b$ and $|a + b|$.

$a \cdot b = |a||b|\cos 150° = 6 \times 4 \times -\frac{\sqrt{3}}{2} = -12\sqrt{3}$
$|a + b|^2 = |a|^2 + 2a \cdot b + |b|^2 = 36 + -24\sqrt{3} + 16 = 52 - 24\sqrt{3}$
$|a + b| = \sqrt{52 - 24\sqrt{3}}$
Answer: $a \cdot b = -12\sqrt{3}$, $|a + b| = \sqrt{52 - 24\sqrt{3}}$

3 Poisson over different intervals (CIE)

Customers enter a shop at random at an average of 4 per hour.
Find the probability that in a 30-minute period there are (a) zero events (b) at least 3.
(c) Find the probability that at least one occurs in each of two separate 30-minute periods.

$\lambda = 4 \times \frac{30}{60} = 2$, $X \sim Po(2)$
(a) $P(X = 0) = e^{-2} = 0.135$
(b) $P(X \geq 3) = 1 - P(X \leq 2) = 0.323$
(c) $(1 - 0.135)^2 = 0.748$
Answer: (a) 0.135 (b) 0.323 (c) 0.748

4 Arc length and sector area (All boards)

A sector of a circle has area 96.8 cm^2 and angle 1.6 radians.
Find the radius, the arc length and the perimeter of the sector.

Area $= \frac{1}{2}r^2\theta$: $96.8 = \frac{1}{2}r^2 \times 1.6$, so $r^2 = 121 = 11^2$
$r = 11$ cm; arc length $s = r\theta = 11 \times 1.6 = 17.6$ cm
Perimeter $= 2r + s = 22 + 17.6 = 39.6$ cm
Answer: r = 11 cm, arc 17.6 cm, perimeter 39.6 cm

5 Normal approximation to the Poisson (CIE)

$X \sim Po(18)$. Use a suitable approximation to find $P(X \leq 25)$, to 3 significant figures.

$\lambda > 15$, so $Y \sim N(18, 18)$ approximately
Continuity correction: $P(Y < 25.5)$
$z = 1.77$, so probability = 0.961
Answer: 0.961

6 Integration by parts (All boards)

Use integration by parts, with $u = \ln x$ and $\frac{dv}{dx} = 1$, to find the exact value of

$\int_1^3 \ln x \, dx$.

$\int \ln x \, dx = x \ln x - \int x \times \frac{1}{x} \, dx = x \ln x - x$
$[x \ln x - x]$ from 1 to 3
$= 3 \ln 3 - 2$
Answer: $3 \ln 3 - 2$

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7 Recurrence relations: limits (All boards)

$u_1 = 40$ and $u_{n+1} = \frac{1}{2} u_n + 10$.

(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.01$.

(a) $L = \frac{1}{2} L + 10$ gives $L = \frac{10}{1 - \frac{1}{2}} = 20$

(b) $u_n - L = (u_1 - L)\, r^{n-1} = 20 \times \left(\frac{1}{2}\right)^{n-1}$

$|20| \times \left(\frac{1}{2}\right)^{n-1} < \frac{1}{100}$

Check: n = 11 gives 0.01953, n = 12 gives 0.00977

Answer: (a) L = 20 (b) n = 12

8 Beam about to tilt (AQA, Edexcel, OCR, MEI)

A uniform plank AB (mass 10 kg, length 4 m) rests horizontally on supports at C and D, where AC = 1.5 m and DB = 1 m. A 4 kg particle is placed at E, AE = x m, and the plank is about to tilt about C. Find x.

About to tilt about C, so the reaction at D is zero and E lies between A and C.
Moments about C: $4g \times (1.5 - x) = 10g \times (2 - 1.5)$
$4(1.5 - x) = 5$
$x = 1.5 - 1.25$

Answer: x = 0.25

9 Tangent conditions (All boards)

Find the set of values of k for which the line $y = 2x + k$ does not meet the curve $y = 2x^2 - 4x + 5$.

$2x^2 - 4x + 5 = 2x + k$ gives $2x^2 - 6x + (5 - k) = 0$
Discriminant: $(-6)^2 - 4 \times 2(5 - k) = -4 + 8k$
No intersection: discriminant < 0

Answer: $k < \frac{1}{2}$

10 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 4 m s^{-1}. 8 s later car B leaves P from rest, in the same direction, with acceleration 2 m s^{-2}. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 8) seconds.
B: $\frac{1}{2} \times 2 \times s^2 = 1\, s^2$ A: $4(s + 8)$
$1\, s^2 - 4\, s - 32 = 0$ so $s = 8$ ($s > 0$)
(b) $PQ = 1 \times 8^2 = 64$
(c) $v = 0 + at = 2 \times 8 = 16$

Answer: (a) 8 s (b) 64 m (c) 16 m s^{-1}