Day 18 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Quadratic from its graph (All boards)

The diagram shows the curve with equation y = f(x), where f is a quadratic.
The turning point is marked, and the curve crosses the y-axis as shown.
Find f(x), giving it in the form $ax^2 + bx + c$.

Turning point (3, 3): $f(x) = a(x - 3)^2 + 3$
$f(0) = -15$: $a(3)^2 + 3 = -15$
$a = -2$, then expand
Answer: $f(x) = -2x^2 + 12x - 15$

2 Differentiating a^x (All boards)

$y = 2 \times 5^x$.
The gradient of the curve at P is 10 ln 5. Find the x-coordinate of P.

$\frac{dy}{dx} = 2 \ln 5 \times 5^x$
$2 \ln 5 \times 5^x = 10 \ln 5$, so $5^x = 5$
$x = 1$, so $x = 1$
Answer: $x = 1$

3 Modulus equations (All boards)

Solve $|x + 2| = -2x - 7$.

Case 1: $x + 2 = -2x - 7$ gives $x = -3$
Case 2: $x + 2 = -(-2x - 7)$ gives $x = -5$
Check $x = -3$: right-hand side $= -1 < 0$, so reject
Check $x = -5$: right-hand side = 3 (valid)
Answer: $x = -5$

4 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate $\int_0^2 3(2^x)\,dx$, giving your answer to 3 s.f.
State, with a reason, whether your estimate is an overestimate or an underestimate.

$h = \frac{1}{2}$; y-values at $x = 0, \frac{1}{2}, 1, \frac{3}{2}, 2$: 3.0000, 4.2426, 6.0000, 8.4853, 12.0000

$T = \frac{1}{4}[3.0000 + 12.0000 + 2(4.2426 + 6.0000 + 8.4853)]$

T = 13.1
The curve is convex ($y'' > 0$) on this interval, so the chords lie above the curve: overestimate
Answer: 13.1, an overestimate

5 Newton's second law with a time-varying force (AQA, Edexcel, OCR, MEI)

A 3 kg particle moves on a smooth horizontal line. At time t s a single force of $(15 - 3t)$ N acts along the line. When t = 0 its speed is 0 m s^{-1}. Find (a) its speed when t = 3, (b) its greatest speed.

F = ma: $a = (-3t + 15) \div 3$
$v = \int a\,dt = 5t - 0.5t^2 + c$; v = 0 when t = 0
(a) v(3) = 10.5
(b) Greatest speed when a = 0: t = 5
v(5) = 12.5
Answer: (a) 10.5 m s^{-1} (b) 12.5 m s^{-1}

6 Normal approximation to the binomial (All boards)

$X \sim B(200, 0.4)$. Use a normal approximation to estimate $P(X \geq 86)$ to 3 significant figures.

np = 80, npq = 48, so $Y \sim N(80, 48)$
With continuity correction: $P(Y > 85.5)$
$z = \frac{85.5 - 80}{6.93} = 0.794$, $P(Z > 0.794) = 0.214$
Answer: 0.214

7 Implicit differentiation: products and cubes (All boards)

A curve has equation $3x^2y + 3y^3 + 2x = -4$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at $(1, -1)$.

Differentiate: $6xy + 3x^2\frac{dy}{dx} + 9y^2\frac{dy}{dx} + 2 = 0$

$\frac{dy}{dx} = -\frac{6xy + 2}{3x^2 + 9y^2}$

At $(1, -1)$: $\frac{dy}{dx} = \frac{1}{3}$, so the normal has gradient -3

Answer: $\frac{dy}{dx} = -\frac{6xy + 2}{3x^2 + 9y^2}$; normal gradient -3

8 Tangent conditions (All boards)

Find the set of values of k for which the line $y = x + k$ meets the curve $y = 2x^2 - 2x + 5$ at two distinct points.

$2x^2 - 2x + 5 = x + k$ gives $2x^2 - 3x + (5 - k) = 0$

Discriminant: $(-3)^2 - 4 \times 2(5 - k) = -31 + 8k$

Two distinct points: discriminant > 0

Answer: $k > \frac{31}{8}$

9 Particles on two smooth inclined planes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest. Find (a) the magnitude of the acceleration and which particle moves down, (b) the tension. Take g = 9.8. Give answers to 3 s.f. if not exact.

A: weight component along plane = 5 g sin 30°; B: 6 g sin 60°

B moves down, A up (6 sin 60° > 5 sin 30°): F = ma for the system

6 g sin 60° − 5 g sin 30° = (5 + 6)a

(a) a = 2.40

(b) For A: T − 5 g sin 30° = 5 a, so T = 36.5

Answer: (a) 2.40 m s^{-2}, B moves down (b) 36.5 N

10 Continuous random variable: cumulative distribution and median (CIE)

The random variable X has pdf $f(x) = \frac{1}{18}(6 - x)$ for $0 \le x \le 6$, and 0 otherwise.

(a) Find the cumulative distribution function F(x) for $0 \le x \le 6$.

(b) Find P(X > 3). (c) Find the median of X to 3 s.f.

(a) $F(x) = \int_0^x f(t)\,dt = \frac{2x}{6} - \frac{x^2}{36}$

(b) $P(X > 3) = 1 - F(3) = \frac{1}{4}$

(c) $F(m) = \frac{1}{2}$: $\frac{2m}{6} - \frac{m^2}{36} = \frac{1}{2}$, so $m = 6(1 - \frac{1}{\sqrt{2}})$

m = 1.76

Answer: (b) $\frac{1}{4}$ (c) 1.76