Day 179 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arc length and sector area (All boards)

A sector of a circle of radius 8 cm has angle 45°.
Find, in terms of π, the arc length, the area and the perimeter of the sector.

$\theta = 45° = \frac{\pi}{4}$ radians

Arc length = $r\theta = 8 \times \frac{\pi}{4} = 2\pi$ cm

Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times \frac{\pi}{4} = 8\pi$ cm^2

Perimeter = 2r + arc = $2\pi + 16$ cm

Answer: arc 2π cm, area 8π cm^2, perimeter $2\pi + 16$ cm

2 Work-energy on a rough slope (CIE)

A 5 kg block is released from rest on a rough plane inclined at α, $\tan\alpha = \frac{4}{3}$, $\mu = 0.5$, and slides 10 m down a line of greatest slope. Use work and energy to find (a) the work done against friction, (b) its speed after 10 m. Take g = 10.

R = 5 g cos α = 30 N, F = μR = 15 N
(a) Work done against friction = F × d = 15 × 10
(b) Loss of PE = 5 g × 10 sin α = 400 J
Work-energy: ½×5×v^2 = 400 − 150 = 250
v^2 = 100

Answer: (a) 150 J (b) 10 m s^{-1}

3 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2 x + 6\sin x - 2 = 0$
for $0° \le x \le 180°$.

Use $\cos^2 x \equiv 1 - \sin^2 x$: $2\sin^2 x - 6\sin x = 0$
Solving the quadratic: sin x = 0 or sin x = 3
sin x = 0: x = 0°, 180°
sin x = 3 has no solutions (|sin x| ≤ 1)

Answer: x = 0°, 180°

4 Modulus graph and a line (All boards)

The diagram shows the graphs of $y = |3x - 6|$ and $y = -x + 6$.
The graphs meet at A and B.
Find the coordinates of A and B.

$|3x - 6| = -x + 6$
Solve $3x - 6 = \pm(-x + 6)$.
x = 0 and x = 3 (both checked in the original equation)
y = 6 and y = 3

Answer: A(0, 6), B(3, 3)

5 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 2 kg particle rests on a rough plane inclined at α, $\tan\alpha = \frac{3}{4}$, $\mu = 0.4$. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

sin α = 0.6, cos α = 0.8, R = 2 g cos α = 15.68 N
Limiting friction = μR = 6.272 N
P smallest: friction acts up the slope: P + μR = 2 g sin α, P = 11.76 − 6.272
P greatest: friction acts down the slope: P = 11.76 + 6.272

Answer: 5.488 N ≤ P ≤ 18.0 N

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6 Venn diagram with algebra (All boards)

The Venn diagram shows the numbers of 44 students in sets A and B.
(a) Find the value of x.
(b) A student is chosen at random. Find P(A | B′).

(a) Total: (2x + 3) + x + 2x + 1 = 44
5x + 4 = 44, so x = 8
(b) Regions: 19, 8, 16, 1
Not in B: 19 + 1 = 20, of which in A: 19
Answer: (a) x = 8 (b) $\frac{19}{20}$

7 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line 3x + 4y = k is a tangent to the circle
$(x - 3)^2 + (y + 3)^2 = 9$.
Find the two possible values of k.

Centre (3, −3), radius 3
Distance from centre to the line = radius:
$\frac{|-3 - k|}{5} = 3$
$|-3 - k| = 15$
Answer: k = −18 or k = 12

8 Double angle equations (All boards)

Solve $\cos 2\theta + 2\sin\theta = 1$
for $-180° \le \theta \le 180°$.

$\cos 2\theta \equiv 1 - 2\sin^2\theta$: $2\sin^2\theta - 2\sin\theta = 0$
Solve the quadratic: $\sin\theta = 1$ or $\sin\theta = 0$
$\sin\theta = 1$: $\theta = 90°$
$\sin\theta = 0$: $\theta = -180°, 0°, 180°$
Answer: θ = −180°, 0°, 90°, 180°

9 Sum to infinity: two possible ratios (All boards)

The second term of a geometric series is 8 and its sum to infinity is 36.
Find the two possible values of the first term.

$ar = 8$ and $\frac{a}{(1 - r)} = 36$
$a = 36(1 - r)$, so $36(1 - r)\,r = 8$
$36r^2 - 36r + 8 = 0$: $r = \frac{1}{3}$ or $r = \frac{2}{3}$
$a = 36(1 - r) = 24$ or 12
Answer: a = 12 or a = 24

10 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

25 values have $\Sigma x = 625$ and $\Sigma x^2 = 16525$.
One value was recorded as 42 but should have been 29.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 625 - 42 + 29 = 612$
$\Sigma x^2 = 16525 - 42^2 + 29^2 = 15602$
Mean = $\frac{612}{25} = 24.5$
sd = $\sqrt{\frac{15602}{25} - (24.5)^2} = 4.98$
Answer: mean 24.5, sd 4.98