Day 178 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ 6 \end{pmatrix}$ and $\begin{pmatrix} k \\ 5 \\ 1 \end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 + 5k + 6 = 0$

$(k + 3)(k + 2) = 0$

$k = -3$ or $k = -2$. For $k = -2$ the first vector is $\begin{pmatrix} -2 \\ -2 \\ 6 \end{pmatrix}$, with length $\sqrt{44} = 2\sqrt{11}$

Answer: $k = -3$ or -2; length $2\sqrt{11}$

2 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 33 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{165}{13}, \frac{396}{13})$. At time T the velocity is $(\frac{165}{13}, \frac{396}{13} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{165}{13})^2 + \frac{396}{13}(\frac{396}{13} - 9.8T) = 0$

$33^2 - 9.8T \times \frac{396}{13} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 33 \div (9.8 \times \frac{12}{13}) = \frac{715}{196}$

(b) $h = \frac{396}{13} \times \frac{715}{196} - 4.9 \times \frac{715}{196}^2 = \frac{10285}{224}$

Answer: (a) T = 3.65 s (b) 45.9 m

3 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the mass (g), is normally distributed with mean 63 and standard deviation 5.

Find (a) P(X > 57.5) (b) P(57.5 < X < 60.5), each to 3 s.f.

(a) $z = \frac{57.5 - 63}{5} = -1.10$, $P(Z > -1.10) = 0.864$

(b) $z_1 = -1.10$, $z_2 = \frac{60.5 - 63}{5} = -0.500$

$P(57.5 < X < 60.5) = \Phi(-0.500) - \Phi(-1.10) = 0.173$

Answer: (a) 0.864 (b) 0.173

4 Completing the square (All boards)

$f(x) = 3x^2 + 18x + 25$.

(a) Write f(x) in the form $a(x + p)^2 + q$.

(b) State the minimum value of f(x) and the value of x at which it occurs.

(c) Solve f(x) = 0, giving surds in simplest form.

$f(x) = 3(x + 3)^2 - 2$

(b) minimum value -2 when $x = -3$

(c) $3(x + 3)^2 = 2$, so $(x + 3)^2 = \frac{2}{3}$

$x + 3 = \pm\sqrt{\frac{2}{3}}$

Answer: (a) $3(x + 3)^2 - 2$ (b) minimum -2 at $x = -3$ (c) $x = -3 \pm \frac{\sqrt{6}}{3}$

5 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 900 kg car tows a 300 kg trailer by a light tow-bar on a horizontal road. Driving force 1230 N; resistances 500 N (car), 250 N (trailer). Find (a) the acceleration and the tow-bar tension. At 15 m s^{-1} the tow-bar breaks. (b) How far does the trailer then travel?

Whole system: $1230 - 500 - 250 = 1200\,a$, so $a = 0.4$

Trailer: $T - 250 = 300 \times 0.4$, so $T = 250 + 300 \times 0.4 = 370$

(b) After the break the trailer decelerates at $250 \div 300 = \frac{5}{6}$ m s^{-2}

$0 = 15^2 - 2 \times \frac{5}{6} \times s$, so $s = 135$

Answer: (a) 0.4 m s^{-2}, 370 N (b) 135 m

6 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $xy - y^2 - 4y = 12$.

Show that P(−3, −3) lies on the curve and find the equation of the tangent at P in the form $ax + by + c = 0$ with integer a, b, c.

Differentiate (product rule on the xy term): $(y) + (x - 2y - 4)\frac{dy}{dx} = 0$

At P(−3, −3): $-3 + (-1)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -3$

Tangent: $y + 3 = (-3)(x + 3)$, i.e. $3x + y + 12 = 0$

Answer: 3x + y + 12 = 0

7 Smallest number of trials (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a seed germinates is 0.03 on each independent trial.

Find the least number of trials needed so that the probability of this happening at least once is greater than 0.995.

P(at least one) $= 1 - 0.97^n > 0.995$

$0.97^n < 0.005$

$n > \frac{\log 0.005}{\log 0.97} = 174$

Least integer $n = 174$

Answer: 174

8 Recurrence relations: limits (All boards)

$u_1 = 0$ and $u_{n+1} = \frac{1}{2} u_n + 4$.

(a) Find the limit L of the sequence as $n \to \infty$.

(b) Find the least n for which $|u_n - L| < 0.1$.

(a) $L = \frac{1}{2} L + 4$ gives $L = \frac{4}{1 - \frac{1}{2}} = 8$

(b) $u_n - L = (u_1 - L)\, r^{n-1} = (-8) \times \left(\frac{1}{2}\right)^{n-1}$

$|-8| \times \left(\frac{1}{2}\right)^{n-1} < \frac{1}{10}$

Check: n = 7 gives 0.12500, n = 8 gives 0.06250

Answer: (a) L = 8 (b) n = 8

9 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

A circular ripple has area A cm^2 increasing at a constant 6 $cm^2 s^{-1}$.

Find the rate at which the circumference is increasing when the radius is 3 cm.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$ and $\frac{dr}{dt} = \frac{6}{2\pi r}$

$C = 2\pi r$, so $\frac{dC}{dt} = 2\pi \times \frac{dr}{dt} = 2\pi \times \frac{6}{2\pi \times 3} = 2$

Answer: 2 cm/s

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10 Combined inequalities (All boards)

(a) Solve $3x - 5 < 6$.

(b) Solve $x^2 - x - 6 \geq 0$.

(c) Hence find the values of x satisfying both inequalities.

(a) $3x - 5 < 6$ so $x < \frac{11}{3}$

(b) $(x + 2)(x - 3) \geq 0$: critical values -2 and 3

(b) gives $x \leq -2$ or $x \geq 3$

(c) Overlap of the two sets.

Answer: (a) $x < \frac{11}{3}$ (b) $x \leq -2$ or $x \geq 3$ (c) $x \leq -2$ or $3 \leq x < \frac{11}{3}$