Day 177 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 2nd term of an arithmetic sequence is 12 and the 9th term is −9.
Find the first term and common difference, and the sum of the first 15 terms.

$u_9 - u_2 = 7\,d = -9 - 12$ so $d = -3$

$a + 1\,d = 12$ so $a = 15$

$S_{15} = \frac{15}{2}(2 \times 15 + 14 \times (-3))$

Answer: $a = 15$, $d = -3$, $S_{15} = -90$

2 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 500 kg carries a person of mass 60 kg. The cable tension is 5264 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 560 kg), taking upwards as positive: $T - 560\,g = 560\,a$

$5264 - 5488 = 560\,a$, so $a = -0.4$

(a) a is negative, so the acceleration is 0.4 m s^{-2} downwards

(b) Person: $R - 60\,g = 60\,a$, so $R = 60(9.8 - 0.4)$

Answer: (a) 0.4 m s^{-2} downwards (b) 564 N

3 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $x^2 - xy + 2y^2 = 63$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{2x - y}{-x + 4y}$

Horizontal tangent: $2x - y = 0$, so $y = 2x$

Substitute: $7x^2 = 63$, so $x^2 = 9$ and $x = \pm 3$

Points: (3, 6) and (−3, −6)

Answer: (3, 6) and (−3, −6)

4 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(−9, 8) and B(15, −2) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(−9, −2) lies on the circle.

Centre = midpoint of AB = (3, 3)

$r^2 = (-12)^2 + (5)^2 = 169$

$(x - 3)^2 + (y - 3)^2 = 169$: $x^2 + y^2 - 6x - 6y - 151 = 0$

(b) $CM^2 = (-12)^2 + (-5)^2 = 169 = r^2$

Answer: (a) $x^2 + y^2 - 6x - 6y - 151 = 0$

5 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 6 kg is pushed at constant speed along rough horizontal ground, $\mu = 0.2$, by a force of P N at angle θ below the horizontal, $\tan\theta = \frac{3}{4}$. Find P and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

Constant speed, so the resultant force is zero. $\sin\theta = 0.6$, $\cos\theta = 0.8$

Vertical: $R = 6\,g + P\sin\theta$

Horizontal: $P\cos\theta = \mu R = 0.2(6\,g + P\sin\theta)$

$P(0.8 - 0.12) = 0.2 \times 6\,g$

$P = 17.3$, $R = 69.2$

Answer: $P = 17.3$ N, $R = 69.2$ N

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6 Recurrence relations: unknown first term (All boards)

A sequence is defined by $u_1 = k$ and $u_{n+1} = -3u_n - 1$.

Given that $u_3 = -7$, find (a) the value of k, (b) $\sum_{r=1}^{4} u_r$.

$u_2 = -3k - 1$, $u_3 = (-3)(-3k - 1) - 1$
$9k + 2 = -7$ so $k = -1$
Terms: $-1, 2, -7, 20$
Sum = 14
Answer: (a) $k = -1$ (b) 14

7 One-tailed binomial hypothesis test (AQA, Edexcel, OCR, MEI, Eduqas)

A claim is that the probability p that a seed germinates is 0.3. In a sample of 19, the number of successes is 12. Test at the 1% level whether p has increased.

$H_0: p = 0.3$, $H_1: p > 0.3$; $X \sim B(19, 0.3)$
$P(X \geq 12) = 1 - P(X \leq 11) = 0.00282$
$0.00282 < 0.01$
Reject H_0: there is evidence that p has increased.
Answer: p-value 0.00282; reject H_0

8 Implicit differentiation: products and cubes (All boards)

A curve has equation $x^2y + y^3 - 3x = 7$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at (1, 2).

Differentiate: $2xy + x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} - 3 = 0$

$$\frac{dy}{dx} = -\frac{2xy - 3}{x^2 + 3y^2}$$

At (1, 2): $\frac{dy}{dx} = -\frac{1}{13}$, so the normal has gradient 13

Answer: $\frac{dy}{dx} = -\frac{2xy - 3}{x^2 + 3y^2}$; normal gradient 13

9 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(21, 0.1)$.

Find the smallest value of c such that $P(X \geq c) < 0.1$.

$P(X \geq 5) = 1 - P(X \leq 4) = 0.0522$
$P(X \geq 4) = 1 - P(X \leq 3) = 0.152$
$0.0522 < 0.1$ but $0.152 > 0.1$
Answer: c = 5

10 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

$l_1: \mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}$

$l_2: \mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $-1 + 3\lambda = -1 + 3\mu$; $0 + 2\lambda = 0 - 2\mu$; $2 - 2\lambda = 2 + 0\mu$
Solving: $\lambda = 0$, $\mu = 0$, and all three equations are satisfied
Point of intersection = $(-1, 0, 2)$
Answer: Intersect at $(-1, 0, 2)$