Day 176 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Poisson probabilities (CIE)

Emails arrive at a mean rate of 3 per minute, as a Poisson process.
Find the probability that in one such period there are
(a) exactly 4 (b) at least 4 (c) fewer than 4 (3 s.f.).

$X \sim Po(3)$

(a) $P(X = 4) = e^{-3}\, 3^4/24! = 0.168$
(b) $1 - P(X \leq 3) = 1 - 0.647 = 0.353$
(c) $P(X \leq 3) = 0.647$
Answer: (a) 0.168 (b) 0.353 (c) 0.647

2 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has constant acceleration $(-\mathbf{i} + 4\mathbf{j})$ m s^{-2}. When t = 0, P is at $(2\mathbf{i} + 5\mathbf{j})$ m moving with velocity $(2\mathbf{i} + 2\mathbf{j})$ m s^{-1}.
Find (a) when P moves parallel to $\mathbf{j}$, (b) its position vector then.

$\mathbf{v} = \mathbf{u} + \mathbf{at} = (2\mathbf{i} + 2\mathbf{j}) + t(-\mathbf{i} + 4\mathbf{j})$
(a) Parallel to $\mathbf{j}$: the $\mathbf{i}$ component is zero: $2 - t = 0$, so $t = 2$
(b) $\mathbf{r} = \mathbf{r}_0 + \mathbf{ut} + \frac{1}{2}\mathbf{at}^2$
$= (2\mathbf{i} + 5\mathbf{j}) + 2(2\mathbf{i} + 2\mathbf{j}) + 2(-\mathbf{i} + 4\mathbf{j})$
$= 4\mathbf{i} + 17\mathbf{j}$
Answer: (a) t = 2 (b) r = (4i + 17j) m

3 Arc length and sector area (All boards)

A sector of a circle has area 100 cm^2 and angle 2 radians.
Find the radius, the arc length and the perimeter of the sector.

Area $= \frac{1}{2}r^2\theta$: $100 = \frac{1}{2}r^2 \times 2$, so $r^2 = 100 = 10^2$
r = 10 cm; arc length $s = r\theta = 10 \times 2 = 20$ cm
Perimeter $= 2r + s = 20 + 20 = 40$ cm
Answer: r = 10 cm, arc 20 cm, perimeter 40 cm

4 Inverse of a quadratic function (All boards)

$f(x) = x^2 - 2x + 4$, $x \geq 1$.
Find $f^{-1}(x)$ and state its domain.

$f(x) = (x - 1)^2 + 3$
Let $y = f(x)$: $(x - 1)^2 = y - 3$
$x \geq 1$, so take the positive root: $x = 1 + \sqrt{y - 3}$
Answer: $\mathbf{f^{-1}(x) = 1 + \sqrt{x - 3}}$, $\mathbf{x \geq 3}$

5 Index equations solved with logarithms (All boards)

Solve $5^{2x} = 3^{2x-2}$.
Give your answer to 3 significant figures.

Take logs: $2x \ln 5 = (2x - 2) \ln 3$
Collect x terms: $x(2 \ln 5 - 2 \ln 3) = -2 \ln 3$
$x = \dfrac{-2 \ln 3}{2 \ln 5 - 2 \ln 3}$
$x = -2.15$
Answer: x = −2.15

6 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $5x + 12y = k$ is a tangent to the circle
$(x - 2)^2 + (y - 2)^2 = 4$.
Find the two possible values of k.

Centre (2, 2), radius 2
Distance from centre to the line = radius:
$\frac{|34 - k|}{13} = 2$
$|34 - k| = 26$
Answer: k = 8 or k = 60

7 Equations with cosec x and cot x (All boards)

Solve $3\text{cosec}^2 x - 9\cot x - 15 = 0$
for $0° < x < 180°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $3\cot^2 x - 9\cot x - 12 = 0$
$\cot x = -1$ or $\cot x = 4$
$\cot x = -1$ means $\tan x = -1$: $x = 135.0°$
$\cot x = 4$ means $\tan x = \frac{1}{4}$: $x = 14.0°$
Answer: x = 14.0°, 135.0°

8 Smooth incline: passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected up a line of greatest slope of a smooth plane inclined at α. It passes A, 29.4 m from the start, after 3 s and again after 5 s. Find (a) the speed of projection, (b) α to 1 d.p. Take g = 9.8.

Deceleration $a = g \sin \alpha$ (unknown). $s = ut - \frac{1}{2}at^2$ at both times:
$u\times3 - \frac{1}{2}a\times3^2 = u\times5 - \frac{1}{2}a\times5^2 = 29.4$
Subtracting: $u(5 - 3) = \frac{1}{2}a(25 - 9)$, so $u = \frac{1}{2}a(8)$
Substituting: $\frac{1}{2}a(8)(3) - \frac{1}{2}a\times3^2 = 29.4$, so $\frac{1}{2}a\times15 = 29.4$, $a = 3.92$
(a) $u = 1.96\times8 = 15.68$
(b) $9.8 \sin \alpha = 3.92$, $\sin \alpha = 0.4$, $\alpha = 23.6°$
Answer: (a) 15.68 m s^{-1} (b) 23.6°

9 Discrete random variable with unknown constant (Edexcel, OCR, MEI, Eduqas, CIE)

The random variable X has $P(X = x) = k(9 - x)$ for x = 2, 3, 4, 5, 6.

(a) Show that $k = \frac{1}{25}$. (b) Find E(X) and Var(X).

(c) Find $P(X \geq 3)$.

(a) $\Sigma P(X = x) = 1$: $7 + 6 + 5 + 4 + 3 = 25$, so $25k = 1$

(b) $E(X) = 2 \times \frac{7}{25} + 3 \times \frac{6}{25} + 4 \times \frac{1}{5} + 5 \times \frac{4}{25} + 6 \times \frac{3}{25} = \frac{18}{5}$

$E(X^2) = \frac{74}{5}$, so $\text{Var}(X) = \frac{74}{5} - (\frac{18}{5})^2 = \frac{46}{25}$

(c) $P(X \geq 3) = \frac{6}{25} + \frac{1}{5} + \frac{4}{25} + \frac{3}{25} = \frac{18}{25}$

Answer: (b) E(X) = $\frac{18}{5}$, Var(X) = $\frac{46}{25}$ (c) $\frac{18}{25}$

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10 Periodic sequences (All boards)

A sequence is defined by $u_1 = \frac{1}{3}$ and $u_{n+1} = \frac{1}{1 - u_n}$.

(a) Find u_2 and u_3.

(b) Find u_{101} and $\sum_{r=1}^{101} u_r$.

$u_2 = \frac{3}{2}$, $u_3 = -2$, $u_4 = \frac{1}{3}$

$u_4 = u_1$, so the sequence repeats with period 3.

$101 = 3 \times 33 + 2$, so $u_{101} = u_2 = \frac{3}{2}$

Sum of one period = $-\frac{1}{6}$; 33 full periods plus 2 extra term(s)

Answer: (a) $\frac{3}{2}$, -2 (b) $u_{101} = \frac{3}{2}$, sum = $-\frac{11}{3}$