Day 175 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 4.9 m s^{-1} from a point h m above horizontal ground, and hits the ground 4 s later. Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: s = −h, u = 4.9, a = −9.8, t = 4
−h = 4.9×4 − 4.9×4^2, so h = 58.8
(b) At the highest point v = 0: 0 = 4.9^2 − 2×9.8×s
s = 1.225 m above the point of projection
Greatest height above the ground = 58.8 + 1.225
Answer: (a) 58.8 m (b) 60.025 m

2 Triangle on a coordinate grid (All boards)

The diagram shows triangle ABC.
(a) Find the equation of the line through C perpendicular to AB.
(b) Hence find the coordinates of D, the foot of the perpendicular from C to AB, and the area of triangle ABC.

Gradient AB = 0, so gradient of the perpendicular = undefined (vertical)
Through C: x = 3
Line AB: y − 1 = 0(x − −2). Solve to get D = (3, 1)
Area = ½ × AB × CD = 24
Answer: (a) x = 3 (b) D(3, 1), area 24

3 Chain rule and gradients (All boards)

$y = (2x + 3)^{-1}$.

Find $\frac{dy}{dx}$ and the gradient of the curve at x = 1.

Let u = 2x + 3, so $y = u^{-1}$, $\frac{du}{dx} = 2$, $\frac{dy}{du} = -1\,u^{-2}$

$\frac{dy}{dx} = -2(2x + 3)^{-2}$

At x = 1: u = 5, so gradient = $-\frac{2}{25}$

Answer: $\frac{dy}{dx} = -2(2x + 3)^{-2}$; gradient = $-\frac{2}{25}$

4 Sum of multiples in a range (All boards)

Find the sum of all the multiples of 8 that lie strictly between 21 and 316.

First multiple 24, last multiple 312, common difference 8.

Number of terms $n = \frac{312 - 24}{8} + 1 = 37$

$S = \frac{37}{2}(24 + 312) = 6216$

Answer: 6216

5 Double and triple angle values (All boards)

Given that $\cos\theta = \frac{2}{7}$, find the exact value of $\cos 3\theta$.

$\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$
$= (2\cos^2\theta - 1)\cos\theta - 2\sin^2\theta \cos\theta = 4\cos^3\theta - 3\cos\theta$
$= 4 \times \left(\frac{2}{7}\right)^3 - 3 \times \frac{2}{7} = -\frac{262}{343}$

Answer: $-\frac{262}{343}$

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6 Poisson probabilities (CIE)

Flaws occur in a metre of cable at a mean rate of 4 per metre, as a Poisson process.
Find the probability that in one such period there are
(a) exactly 3 (b) at least 3 (c) fewer than 3 (3 s.f.).

X ~ Po(4)

(a) $P(X = 3) = e^{-4}\, 4^3/6! = 0.195$

(b) $1 - P(X \leq 2) = 1 - 0.238 = 0.762$

(c) $P(X \leq 2) = 0.238$

Answer: (a) 0.195 (b) 0.762 (c) 0.238

7 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(3 - 3x)^{-2}$ in ascending powers of x, up to and including x^2.
State the range of x for which the expansion is valid.

$(3 - 3x)^{-2} = 3^{-2}(1 + \frac{-3x}{3})^{-2}$

$3^{-2} = \frac{1}{9}$

Expand $(1 + y)^{-2}$ with $y = \frac{-3x}{3}$

Valid for $|\frac{-3x}{3}| < 1$, i.e. $|x| < 1$

Answer: $\frac{1}{9} + \frac{2}{9}x + \frac{1}{3}x^2$; $|x| < 1$

8 Conditional probability without replacement (All boards)

A bag has 7 black and 6 white counters. Two are taken at random without replacement.
Given that the second counter is black, find the probability that the first was black.

P(black then black) = $\frac{7}{13} \times \frac{6}{12} = \frac{7}{26}$

P(white then black) = $\frac{6}{13} \times \frac{7}{12} = \frac{7}{26}$

P(second black) = $\frac{7}{26} + \frac{7}{26} = \frac{7}{13}$

P(first black | second black) = $\frac{\frac{7}{26}}{\frac{7}{13}}$

$= \frac{1}{2}$

Answer: $\frac{1}{2}$

9 Smooth incline: passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected up a line of greatest slope of a smooth plane inclined at α. It passes A, 58.8 m from the start, after 4 s and again after 6 s. Find (a) the speed of projection, (b) α to 1 d.p. Take g = 9.8.

Deceleration a = g sin α (unknown). $s = ut - \frac{1}{2}at^2$ at both times:
$u\times4 - \frac{1}{2}a\times4^2 = u\times6 - \frac{1}{2}a\times6^2 = 58.8$
Subtracting: $u(6 - 4) = \frac{1}{2}a(36 - 16)$, so $u = \frac{1}{2}a(10)$
Substituting: $\frac{1}{2}a(10)(4) - \frac{1}{2}a\times4^2 = 58.8$, so $\frac{1}{2}a\times24 = 58.8$, a = 4.9
(a) u = 2.45×10 = 24.5
(b) 9.8 sin α = 4.9, sin α = 0.5, α = 30.0°

Answer: (a) 24.5 m s^{-1} (b) 30.0°

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10 Optimisation problems (All boards)

An open-topped box has a square base of side x cm and volume 108 cm^3.

Show that its surface area is $A = x^2 + \frac{432}{x}$, and find the least possible value of A, justifying that it is a minimum.

$h = \frac{108}{x^2}$, so $A = x^2 + 4xh = x^2 + \frac{432}{x}$

$\frac{dA}{dx} = 2x - \frac{432}{x^2} = 0$, so $x^3 = 216$ and $x = 6$

$\frac{d^2A}{dx^2} = 2 + \frac{864}{x^3} > 0$, so a minimum

$A = 36 + \frac{432}{6} = 108$

Answer: A_min = 108 cm^2 (x = 6)