Day 174 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Regression line from summary data (AQA, Edexcel, OCR, MEI, Eduqas)

n = 10 pairs of (x, y): $\Sigma x = 164$, $\Sigma y = 297$, $\Sigma x^2 = 3264$, $\Sigma xy = 5506$.
(a) Find the equation of the regression line of y on x (3 s.f.).
(b) Estimate y when x = 21.

$S_{xx} = 3264 - \frac{164^2}{10} = 574$, $S_{xy} = 5506 - \frac{164 \times 297}{10} = 635$

$b = \frac{S_{xy}}{S_{xx}} = 1.11$

$a = \bar{y} - b\,\bar{x} = 29.7 - (1.11)(16.4) = 11.6$

(b) $y = 11.6 + (1.11)(21) = 34.8$

Answer: (a) $y = 11.6 + 1.11x$ (b) 34.8

2 Change of sign to locate a root (All boards)

$f(x) = x^3 - 3x - 11$.
Show that the equation f(x) = 0 has a root α in the interval [2, 3].

f(x) is continuous on the interval
$f(2) = -9.00 < 0$
$f(3) = 7.00 > 0$
A change of sign, so a root lies in [2, 3]

Answer: Sign change: $f(2) = -9.00$, $f(3) = 7.00$

3 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 5 m s^{-1} and accelerates uniformly. She travels 117.6 m in the next 12 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $117.6 = 5 \times 12 + \frac{1}{2} \times a \times 12^2$, so $a = 0.8$
(b) $v = u + at = 5 + 0.8 \times 12 = 14.6$
(c) For the midpoint, $s = 58.8$: $v^2 = u^2 + 2as = 5^2 + 2 \times 0.8 \times 58.8$
$v^2 = 119.08$

Answer: (a) 0.8 m s^{-2} (b) 14.6 m s^{-1} (c) 10.9 m s^{-1}

4 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix} -5 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$. The point P lies on AB with AP : PB = 3 : 1.

Find the position vector of P and the exact distance OP.

$\overrightarrow{OP} = \overrightarrow{OA} + \frac{3}{4}\overrightarrow{AB} = \begin{pmatrix} -5 \\ -5 \end{pmatrix} + \frac{3}{4}\begin{pmatrix} 8 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$OP = \sqrt{1^2 + (-2)^2} = \sqrt{5} = \sqrt{5}$

Answer: $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, OP = $\sqrt{5}$

5 Tree diagram with conditional probability (All boards)

The tree diagram shows probabilities for events R and S.
(a) Find P(S′).
(b) Find P(R′ | S′).

(a) $P(S') = 0.2 \times 0.8 + 0.8 \times 0.7 = 0.72$

(b) $P(R' \mid S') = \frac{0.8 \times 0.7}{0.72}$

Answer: (a) 0.72 (b) $\frac{7}{9}$

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6 Equation of a reciprocal curve from its graph (All boards)

The diagram shows the curve $y = \frac{k}{x - a} + b$ with asymptotes as shown. Find the values of k, a and b, and the coordinates where the curve crosses the axes.

Asymptotes x = 3, y = 1 so a = 3, b = 1

Through (5, 0): $0 = \frac{k}{2} + 1$

$k = (0 - 1)(2) = -2$

x = 0: $y = \frac{5}{3}$; y = 0: x = 5

Answer: $k = -2$, $a = 3$, $b = 1$; $(0, \frac{5}{3})$ and $(5, 0)$

7 Ships on collision courses (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

At 12:00 ship P has position $(15\mathbf{i} - 30\mathbf{j})$ km and velocity $(-\mathbf{i} + 6\mathbf{j})$ km h^{-1}; ship Q has position $(-5\mathbf{i} + 10\mathbf{j})$ km and velocity $(3\mathbf{i} - 2\mathbf{j})$ km h^{-1}. Show that the ships collide and find the time and position vector of the collision.

At time t hours: $\mathbf{r}_P = (15\mathbf{i} - 30\mathbf{j}) + t(-\mathbf{i} + 6\mathbf{j})$, $\mathbf{r}_Q = (-5\mathbf{i} + 10\mathbf{j}) + t(3\mathbf{i} - 2\mathbf{j})$

Equal when: **i**: $15 + -1\,t = -5 + 3\,t$, so $t = 5$

j: $-30 + 5 \times 6 = 0$

Check with Q: **i**: $-5 + 5 \times 3 = 10$, **j**: $10 + 5 \times (-2) = 0$

Both components agree at t = 5, so they collide at $(10\mathbf{i})$ km

Answer: t = 5 hours (at 17:00); r = (10i) km

8 Cubic inequalities (All boards)

Given that $(x - 5)$ is a factor of $f(x) = x^3 + 2x^2 - 25x - 50$, solve $f(x) < 0$.

Factorise: $f(x) = (x + 5)(x + 2)(x - 5)$

Roots at $x = -5, -2, 5$

Sketch the cubic (positive x^3 coefficient: negative, positive, negative, positive)

Read off where $f(x) < 0$.

Answer: $x < -5$ or $-2 < x < 5$

9 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

A hemispherical bowl of radius 8 cm is filled at 20 $cm^3 s^{-1}$. When the depth is h, $V = \pi(8h^2 - \frac{1}{3}h^3)$.

Find $\frac{dh}{dt}$ when h = 4 cm.

$\frac{dV}{dh} = \pi(16h - h^2) = \pi h(16 - h)$

$\frac{dh}{dt} = \frac{20}{\pi h(16 - h)}$

h = 4: $\frac{dh}{dt} = \frac{20}{\pi \times 4 \times 12} = \frac{5}{12\pi}$ cm s^{-1}

Answer: $\frac{5}{12\pi}$ cm/s

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10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{17x + 6}{(1 + 3x)(1 + 2x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 + 3x)} + \frac{B}{(1 + 2x)}$: A = 1, B = 5

(b) $1(1 + 3x)^{-1} = 1(1 - 3x + 9x^2 ...)$

Add the series for both fractions.

(c) Need $|3x| < 1$ and $|2x| < 1$

Answer: (a) $\frac{1}{(1 + 3x)} + \frac{5}{(1 + 2x)}$ **(b)** $6 - 13x + 29x^2$ **(c)** $|x| < \frac{1}{3}$