Day 172 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(3, −9) and B(3, 1) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(0, 0) lies on the circle.

Centre = midpoint of AB = (3, −4)
$r^2 = (0)^2 + (-5)^2 = 25$
$(x - 3)^2 + (y + 4)^2 = 25$: $x^2 + y^2 - 6x + 8y = 0$
(b) $CM^2 = (-3)^2 + (4)^2 = 25 = r^2$
Answer: (a) $x^2 + y^2 - 6x + 8y = 0$

2 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 9.8 m s^{-1} from a point h m above horizontal ground, and hits the ground 4 s later.
Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: s = −h, u = 9.8, a = −9.8, t = 4
$-h = 9.8 \times 4 - 4.9 \times 4^2$, so h = 39.2
(b) At the highest point v = 0: $0 = 9.8^2 - 2 \times 9.8 \times s$
s = 4.9 m above the point of projection
Greatest height above the ground = 39.2 + 4.9
Answer: (a) 39.2 m (b) 44.1 m

3 Change of sign to locate a root (All boards)

$f(x) = e^x - 4x - 11$.
Show that the equation f(x) = 0 has a root α in the interval [3, 4].

f(x) is continuous on the interval
f(3) = −2.91 < 0
f(4) = 27.6 > 0
A change of sign, so a root lies in [3, 4]
Answer: Sign change: f(3) = −2.91, f(4) = 27.6

4 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

The 76 items in a list are numbered 1 to 76.
A systematic sample of 19 items is taken, starting with a random item among the first few.
The first item selected is number 2.
Find the sampling interval and the number of the last item selected.

Interval = 76 ÷ 19 = 4
Items selected: 2, 6, 10, ...
Last = 2 + (19 − 1) × 4 = 74
Answer: Interval 4, last item 74

5 Fixed point iteration (All boards)

The equation $e^x = 3x + 8$ has a root α. Use the iteration $x_{n+1} = \ln(3x + 8)$ with $x_0 = 2$
to find x_1, x_2 and x_3 to 4 d.p., and hence the value of α to 2 d.p. (continue until the 2 d.p. value is settled).

$x_1 = \ln(3 \times 2 + 8) = 2.6391$
$x_2 = 2.7674$, $x_3 = 2.7913$
Continuing, the values settle at 2.7967, so α = 2.80
Answer: $x_1 = 2.6391$, $x_2 = 2.7674$, $x_3 = 2.7913$; α = 2.80

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6 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $-3x + 4y = k$ is a tangent to the circle
$(x - 1)^2 + (y + 2)^2 = 25$.
Find the two possible values of k.

Centre $(1, -2)$, radius 5
Distance from centre to the line = radius:
$\frac{|-11 - k|}{5} = 5$
$|-11 - k| = 25$
Answer: $k = -36$ or $k = 14$

7 Hypothesis test for a normal mean (All boards)

The mean fill volume of bottles (ml) is normally distributed with standard deviation 10; it was previously 30.
A random sample of 9 has mean 21.3.
Test, at the 10% level, whether the mean has changed.

$H_0: \mu = 30$, $H_1: \mu \neq 30$

$\bar{X} \sim N(30, \frac{10^2}{9})$, $z = \frac{21.3 - 30}{\frac{10}{3}} = -2.61$

p-value = 0.00905 (two-tailed) < 0.1
Reject H_0: evidence that the mean has changed.
Answer: $z = -2.61$, $p = 0.00905$; reject H_0

8 Modulus inequalities (All boards)

Solve $|x + 1| > |-2x + 6|$.

Square both sides: $(x + 1)^2 > (-2x + 6)^2$
$((x + 1) - (-2x + 6))((x + 1) + (-2x + 6)) > 0$
$(3x - 5)(-x + 7) > 0$

Critical values $x = \frac{5}{3}$ and $x = 7$; sketch the (positive or negative) quadratic.

Answer: $\frac{5}{3} < x < 7$

9 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 16 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{15}{8}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{128}{17}, \frac{240}{17})$. At time T the velocity is $(\frac{128}{17}, \frac{240}{17} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{128}{17})^2 + \frac{240}{17}(\frac{240}{17} - 9.8T) = 0$

$16^2 - 9.8T \times \frac{240}{17} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 16 \div (9.8 \times \frac{15}{17}) = \frac{272}{147}$

(b) $h = \frac{240}{17} \times \frac{272}{147} - 4.9 \times \frac{272}{147}^2 = \frac{2944}{315}$

Answer: (a) T = 1.85 s (b) 9.35 m

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10 Implicit differentiation: products and cubes (All boards)

A curve has equation $3x^2y + 2y^3 + 2x = 16$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at (0, 2).

Differentiate: $6xy + 3x^2\frac{dy}{dx} + 6y^2\frac{dy}{dx} + 2 = 0$

$\frac{dy}{dx} = -\frac{6xy + 2}{3x^2 + 6y^2}$

At (0, 2): $\frac{dy}{dx} = -\frac{1}{12}$, so the normal has gradient 12

Answer: $\frac{dy}{dx} = -\frac{6xy + 2}{3x^2 + 6y^2}$; normal gradient 12