Day 171 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Smooth incline: passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected up a line of greatest slope of a smooth plane inclined at α. It passes A, 19.6 m from the start, after 2 s and again after 8 s. Find (a) the speed of projection, (b) α to 1 d.p. Take g = 9.8.

Deceleration a = g sin α (unknown). $s = ut - \frac{1}{2}at^2$ at both times:
$u\times2 - \frac{1}{2}a\times2^2 = u\times8 - \frac{1}{2}a\times8^2 = 19.6$
Subtracting: $u(8 - 2) = \frac{1}{2}a(64 - 4)$, so $u = \frac{1}{2}a(10)$
Substituting: $\frac{1}{2}a(10)(2) - \frac{1}{2}a\times2^2 = 19.6$, so $\frac{1}{2}a\times16 = 19.6$, $a = 2.45$
(a) $u = 1.225\times10 = 12.25$
(b) $9.8 \sin\alpha = 2.45$, $\sin\alpha = 0.25$, $\alpha = 14.5°$

Answer: (a) 12.25 m s^{-1} (b) 14.5°

2 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 + 6x - 4y + 3 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(−1, 3) lies inside or outside the circle.

Complete the square: $(x + 3)^2 + (y - 2)^2 = 10$
Centre (−3, 2), radius $\sqrt{10}$
Distance^2 from centre to P = $2^2 + 1^2 = 5$
Compare with $r^2 = 10$

Answer: (a) centre (−3, 2), radius $\sqrt{10}$ (b) P is inside the circle

3 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

Water pours into a cone, vertex down, at 10 cm^{3}s^{-1}. The surface radius is always $\frac{h}{2}$.

Find the rate at which the depth is rising when h = 2 cm.

$V = \frac{1}{3}\pi(h/2)^2h = \frac{\pi h^3}{12}$, so $\frac{dV}{dh} = \frac{\pi h^2}{4}$

$$\frac{dh}{dt} = \frac{dV}{dt \div dV/dh} = \frac{10 \times 4}{\pi h^2}$$

h = 2: $\frac{dh}{dt} = \frac{10}{\pi}$ cm s^{-1}

Answer: $\frac{10}{\pi}$ cm/s

4 Sigma notation (All boards)

Given that $\sum_{r=1}^{n}(4r - 3) = 780$, find the value of n.

$$\sum_{r=1}^{n}(4r - 3) = 4 \times \frac{n(n + 1)}{2} - 3n$$

$2n^2 - n = 780$
$4n^2 - 2n - 1560 = 0$
Solve and reject the negative root: n = 20
Answer: 20

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5 Bayes' theorem in context (All boards)

0.5% of people have a fault. The inspection test is positive for 80% of those who have it and for 2% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the fault (3 s.f.).

(a) P(+) = 0.005 × 0.8 + 0.995 × 0.02
= 0.004 + 0.0199 = 0.0239

(b) P(has it | +) = $\frac{0.005 \times 0.8}{0.0239}$

= 0.167

Answer: (a) 0.0239 (b) 0.167

6 Area and perimeter of a segment (exact) (All boards)

The circle has centre O and radius 10 cm. Angle AOB = $\frac{\pi}{2}$.

Find the exact area of the shaded segment.

Sector area = $\frac{1}{2}r^2\theta = 25\pi$; triangle OAB = $\frac{1}{2}r^2\sin\theta = 50$

Segment area = sector − triangle = $25\pi - 50$

Answer: area = $25\pi - 50$ cm^2

7 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (3 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (1 kg), hanging 1.2 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: 1 g − T = 1 a
A: T = 3 a
Adding: 1 g = 4 a, so a = 2.45
(a) T = 3 × 2.45 = 7.35
(b) $v^2 = 0 + 2\times2.45\times1.2 = 5.88$

Answer: (a) 7.35 N (b) 2.42 m s^{-1}

8 Partial fractions and integration (All boards)

(a) Express $\frac{-3x - 13}{(x + 1)(x + 6)}$ in partial fractions.

(b) Hence find $\int_0^1 \frac{-3x - 13}{(x + 1)(x + 6)}\,dx$, giving the answer in the form ln k.

$-3x - 13 \equiv A(x + 6) + B(x + 1)$: x = −6 gives A = −2; x = −1 gives B = −1

$I = [-2\ln(x + 1) - \ln(x + 6)]_0^1$

$= -2\ln\frac{2}{1} - \ln\frac{7}{6}$

$= \ln\frac{3}{14}$

Answer: (a) $\frac{-2}{x + 1} - \frac{1}{x + 6}$; (b) $\ln\frac{3}{14}$

9 Tangent conditions (All boards)

The line y = k − 4x is a tangent to the curve $y = \frac{1}{x}$.

Find the possible values of k.

$\frac{1}{x} = k - 4x$

$1 = kx - 4x^2$, so $4x^2 - kx + 1 = 0$

Tangent: discriminant $k^2 - 4 \times 4 \times 1 = 0$

$k^2 = 16$

Answer: k = 4 or k = −4

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10 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Eggs are defective with probability 0.3, independently. A tray of 7 eggs is rejected if it has at least 2 defective. From 10 such trays, find the probability that exactly 3 are rejected (3 s.f.).

$Y \sim B(7, 0.3)$. $P(\text{rejected}) = P(Y \geq 2) = 1 - 0.329 = 0.671$

$W \sim B(10, 0.671)$ is the number rejected

$P(W = 3) = C(10, 3)(0.671)^3(1 - 0.671)^7 = 0.0152$

Answer: 0.0152