Day 170 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 4 m s^{-1} and accelerates uniformly. She travels 70.4 m in the next 8 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $70.4 = 4\times8 + \frac{1}{2}\times a\times8^2$, so $a = 1.2$
(b) $v = u + at = 4 + 1.2\times8 = 13.6$
(c) For the midpoint, $s = 35.2$: $v^2 = u^2 + 2as = 4^2 + 2\times1.2\times35.2$
$v^2 = 100.48$
Answer: (a) 1.2 m s^{-2} (b) 13.6 m s^{-1} (c) 10.0 m s^{-1}

2 Bayes' theorem in context (All boards)

1% of people have a disease. The screening test is positive for 95% of those who have it and for 2% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the disease (3 s.f.).

(a) $P(+) = 0.01 \times 0.95 + 0.99 \times 0.02$
$= 0.0095 + 0.0198 = 0.0293$
(b) $P(\text{has it} \mid +) = \frac{0.01 \times 0.95}{0.0293}$
$= 0.324$
Answer: (a) 0.0293 (b) 0.324

3 Combined inequalities (All boards)

(a) Solve $2x - 1 < 4$.
(b) Solve $x^2 + 4x - 5 > 0$.
(c) Hence find the values of x satisfying both inequalities.

(a) $2x - 1 < 4$ so $x < \frac{5}{2}$
(b) $(x + 5)(x - 1) > 0$: critical values -5 and 1
(b) gives $x < -5$ or $x > 1$
(c) Overlap of the two sets.
Answer: (a) $x < \frac{5}{2}$ (b) $x < -5$ or $x > 1$ (c) $x < -5$ or $1 < x < \frac{5}{2}$

4 Exponential equations reducing to quadratics (All boards)

Solve $4^x - 16 = 0$.
Give your answer to 3 significant figures if not exact.

$u^2 - 16 = 0$
$(u - 4)(u + 4) = 0$, so $u = 4$ or $u = -4$
2^x is always positive, so reject $u = -4$; $2^x = 4$ gives $x = 2$
Answer: $x = 2$

5 Equation of a reciprocal curve from its graph (All boards)

The diagram shows the curve $y = \frac{k}{x - a} + b$ with asymptotes as shown. Find the values of k, a and b, and the coordinates where the curve crosses the axes.

Asymptotes $x = 1$, $y = -2$ so $a = 1$, $b = -2$
Through (4, 0): $0 = \frac{k}{3} - 2$
$k = (0 + 2)(3) = 6$
$x = 0$: $y = -8$; $y = 0$: $x = 4$
Answer: $k = 6$, $a = 1$, $b = -2$; (0, −8) and (4, 0)

6 Hypothesis test for a normal mean (All boards)

The mean mass of a sack (kg) is normally distributed with standard deviation 6; it was previously 49.
A random sample of 49 has mean 48.4.
Test, at the 10% level, whether the mean has changed.

H_0: $\mu = 49$, H_1: $\mu \neq 49$

$\bar{X} \sim N(49, \frac{6^2}{49})$, $z = \frac{48.4 - 49}{\frac{6}{7}} = -0.700$

p-value = 0.484 (two-tailed) > 0.1
Do not reject H_0: no evidence that the mean has changed.

Answer: $z = -0.700$, $p = 0.484$; do not reject H_0

7 Connected particles (rough table, string slack) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (7 kg) lies on a rough table, $\mu = 0.1$, joined by a light string over a smooth pulley to particle B (3 kg), 1 m above the floor. Released from rest; B lands without rebounding. Find (a) the acceleration of A before B lands, (b) the further distance A moves. Take $g = 9.8$.

B: $3g - T = 3a$; A: $T - 0.1 \times 7g = 7a$
Adding: $(3 - 0.1 \times 7)g = 10a$, so $a = 2.254$
Speed of B and A as B lands: $v^2 = 2 \times 2.254 \times 1 = 4.508$
After B lands, A decelerates at $\mu g = 0.1 \times 9.8 = 0.98$
$0 = v^2 - 2(\mu g)s$, $s = 4.508 \div 1.96$

Answer: (a) 2.254 m s^{-2} (b) 2.3 m

8 Greatest and least values using R-form (All boards)

$g(\theta) = \frac{1}{11 + 6\sin\theta + 8\cos\theta}$

(a) Write $6\sin\theta + 8\cos\theta$ in the form $R\sin(\theta + \alpha)$, with α acute.
(b) Find the greatest and least values of $g(\theta)$ and the smallest positive θ at which the greatest value occurs (1 d.p.).

$R = \sqrt{6^2 + 8^2} = 10$, $\alpha = \tan^{-1}\frac{8}{6} = 53.1°$

Denominator $= 11 + 10\sin(\theta + 53.1°)$, ranging from 1 to 21

Greatest $g = \frac{1}{1}$ (denominator least), least $g = \frac{1}{21}$

Least denominator when $\sin(\theta + 53.1°) = -1$: $\theta + 53.1° = 270°$
$\theta = 216.9°$

Answer: $R = 10$, $\alpha = 53.1°$; greatest $\frac{1}{1}$, least $\frac{1}{21}$; $\theta = 216.9°$

9 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of lg y against lg x for $y = A x^n$.
Find the value of n and the value of A.

$\lg y = \lg A + n \lg x$: gradient n, intercept $\lg A$

$n = \frac{-2 - 0}{4 - 2} = -1$

At (2, 0): $\lg A = 0 - (-1) \times 2 = 2$, so $A = 10^2 = 100$

Answer: $n = -1$, $A = 100$

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10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-8x + 1}{(1 - 2x)(1 + x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - 2x)} + \frac{B}{(1 + x)}$: $A = -2$, $B = 3$

(b) $-2(1 - 2x)^{-1} = -2(1 + 2x + 4x^2 ...)$

Add the series for both fractions.

(c) Need $|2x| < 1$ and $|x| < 1$

Answer: (a) $\frac{-2}{(1 - 2x)} + \frac{3}{(1 + x)}$ **(b)** $1 - 7x - 5x^2$ **(c)** $|x| < \frac{1}{2}$