Day 17 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

$|a| = 2$, $|b| = 7$ and the angle between a and b is 60°.
Find the exact values of $a \cdot b$ and $|a + b|$.

$a \cdot b = |a||b|\cos 60° = 2 \times 7 \times \frac{1}{2} = 7$

$|a + b|^2 = |a|^2 + 2a \cdot b + |b|^2 = 4 + 14 + 49 = 67$

$|a + b| = \sqrt{67}$

Answer: $a \cdot b = 7$, $|a + b| = \sqrt{67}$

2 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 200 kg carries a person of mass 75 kg. The cable tension is 2832.5 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take $g = 9.8$.

Whole system (mass 275 kg), taking upwards as positive: $T - 275\,g = 275\,a$
$2832.5 - 2695 = 275\,a$, so $a = 0.5$
(a) a is positive, so the acceleration is 0.5 m s^{-2} upwards
(b) Person: $R - 75\,g = 75\,a$, so $R = 75(9.8 + 0.5)$

Answer: (a) 0.5 m s^{-2} upwards (b) 772.5 N

3 Combined inequalities (All boards)

(a) Solve $3x + 6 > 5$.
(b) Solve $x^2 - 6x + 5 \geq 0$.
(c) Hence find the values of x satisfying both inequalities.

(a) $3x + 6 > 5$ so $x > -\frac{1}{3}$
(b) $(x - 1)(x - 5) \geq 0$: critical values 1 and 5
(b) gives $x \leq 1$ or $x \geq 5$
(c) Overlap of the two sets.

Answer: (a) $x > -\frac{1}{3}$ (b) $x \leq 1$ or $x \geq 5$ (c) $-\frac{1}{3} < x \leq 1$ or $x \geq 5$

4 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$. The point P lies on AB with AP : PB = 3 : 1.

Find the position vector of P and the exact distance OP.

$\overrightarrow{OP} = \overrightarrow{OA} + \frac{3}{4}\overrightarrow{AB} = \begin{pmatrix} 5 \\ -4 \end{pmatrix} + \frac{3}{4}\begin{pmatrix} -4 \\ 8 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$OP = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

Answer: $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$, OP = $2\sqrt{2}$

5 Arithmetic series: S_n formula (All boards)

The sum of the first n terms of an arithmetic series is $S_n = 3n^2 - 2n$.
(a) Find the first term and the common difference.
(b) Find u_{17}.

$u_1 = S_1 = 1$
$S_2 = 8$, so $u_2 = 7$
$d = u_2 - u_1 = 6$
$u_{17} = 1 + 16 \times 6$

Answer: (a) a = 1, d = 6 (b) 97

6 Discrete random variable with unknown constant (Edexcel, OCR, MEI, Eduqas, CIE)

The random variable X has P(X = x) = k(x + 2) for x = 2, 3, 4, 5.

(a) Show that k = $\frac{1}{22}$. (b) Find E(X) and Var(X).

(c) Find P(X ≥ 3).

(a) Σ P(X = x) = 1: 4 + 5 + 6 + 7 = 22, so 22k = 1

(b) E(X) = $2 \times \frac{2}{11} + 3 \times \frac{5}{22} + 4 \times \frac{3}{11} + 5 \times \frac{7}{22} = \frac{41}{11}$

$E(X^2) = \frac{166}{11}$, so $Var(X) = \frac{166}{11} - (\frac{41}{11})^2 = \frac{145}{121}$

(c) $P(X \geq 3) = \frac{5}{22} + \frac{3}{11} + \frac{7}{22} = \frac{9}{11}$

Answer: (b) E(X) = $\frac{41}{11}$, Var(X) = $\frac{145}{121}$ (c) $\frac{9}{11}$

7 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (2 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (8 kg), hanging 1 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: 8 g − T = 8 a

A: T = 2 a

Adding: 8 g = 10 a, so a = 7.84

(a) T = 2 × 7.84 = 15.68

(b) $v^2 = 0 + 2 \times 7.84 \times 1 = 15.68$

Answer: (a) 15.68 N (b) 3.96 m s^{-1}

8 Tangent through the origin (All boards)

The line through the origin is a tangent to the curve $y = x^2 + 2$.
Find the gradients of the two possible tangents and the points of contact.

Tangent at x = a: $y - (a^2 + 2) = 2a(x - a)$. Through (0, 0): $-a^2 - 2 = -2a^2$

So $a^2 = 2$ and $a = \pm\sqrt{2}$

Gradient $2a = \pm 2\sqrt{2}$

Answer: gradients $\pm 2\sqrt{2}$ at points $(\pm\sqrt{2}, 4)$

9 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

X ~ B(20, p). H_0: p = 0.4 is tested against H_1: p > 0.4 using a 5% significance level.
(a) Find the critical region. (b) State the probability of a Type I error.
(c) Find the probability of a Type II error when p = 0.6 (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: X ≥ 13

(b) Type I error = P(reject H_0 | p = 0.4) = 0.0210

(c) Type II error = P(not in critical region | p = 0.6)

P(X ≤ 12) when p = 0.6: 0.584

Answer: (a) X ≥ 13 (b) 0.0210 (c) 0.584

10 Tangent conditions (All boards)

The line y = k − x is a tangent to the curve $y = \frac{1}{x}$.

Find the possible values of k.

$\frac{1}{x} = k - x$

$1 = kx - x^2$, so $x^2 - kx + 1 = 0$

Tangent: discriminant $k^2 - 4 \times 1 \times 1 = 0$

$k^2 = 4$

Answer: k = 2 or k = −2