Day 168 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Venn diagrams and conditional probability (All boards)

P(A) = 0.6, P(B) = 0.25 and P(A ∩ B) = 0.2.
Find
(a) P(A ∪ B)
(b) P(A′ ∩ B′)
(c) P(A | B)

(a) P(A ∪ B) = P(A) + P(B) − P(A ∩ B) = 0.6 + 0.25 − 0.2 = 0.65
(b) P(A′ ∩ B′) = 1 − P(A ∪ B) = 1 − 0.65 = 0.35
(c) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.25} = 0.8$

Answer: (a) 0.65 (b) 0.35 (c) 0.8

2 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 + 1$ and $g(x) = 3x - 1$.
Solve fg(x) = gf(x).

$fg(x) = (3x - 1)^2 + 1 = 9x^2 - 6x + 2$
$gf(x) = 3(x^2 + 1) - 1 = 3x^2 + 2$
$9x^2 - 6x + 2 = 3x^2 + 2$ so $6x^2 - 6x = 0$

Answer: x = 0 or x = 1

3 Selections with combinations (OCR, MEI, Eduqas, CIE)

A group of 6 boys and 8 girls. A team of 3 is chosen at random.
Find the probability that the team contains at least one of the girls.

Total teams = C(14, 3) = 364
Teams of all boys = C(6, 3) = 20
$P(\text{at least one girl}) = 1 - \frac{20}{364}$
$= \frac{86}{91}$

Answer: $\mathbf{\frac{86}{91}}$

4 Factor theorem and remainder theorem (All boards)

$f(x) = 2x^3 + ax^2 + bx - 16$, where a and b are constants.
(x + 4) is a factor of f(x). The remainder when f(x) is divided by (x + 3) is 8.
(a) Find a and b. (b) Solve f(x) = 0.

f(−4) = 0: −128 + 16a − 4b − 16 = 0
f(−3) = 8: −54 + 9a − 3b − 16 = 8
Solving simultaneously: a = 10, b = 4
$f(x) = 2x^3 + 10x^2 + 4x - 16 = 2(x + 4)(x + 2)(x - 1)$

Answer: (a) a = 10, b = 4 (b) x = −4, −2, 1

5 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (−1, 0, 1), (−2, 0, −1) and (4, 1, 4).
Find angle ABC, in degrees to 1 d.p.

$BA = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, $BC = \begin{pmatrix} 6 \\ 1 \\ 5 \end{pmatrix}$
BA·BC = 16
$|BA| = \sqrt{5}$, $|BC| = \sqrt{62}$
$\cos ABC = \frac{16}{\sqrt{5} \times \sqrt{62}} = 0.9087$, so angle ABC = 24.7°

Answer: 24.7°

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6 Sum of multiples in a range (All boards)

Find the sum of all the multiples of 4 that lie strictly between 54 and 419.

First multiple 56, last multiple 416, common difference 4.

Number of terms $n = \frac{416 - 56}{4} + 1 = 91$

$S = \frac{91}{2}(56 + 416) = 21476$

Answer: 21476

7 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2 x + 10\cot x - 10 = 0$
for $0° < x < 360°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $4\cot^2 x + 10\cot x - 6 = 0$

$\cot x = \frac{1}{2}$ or $\cot x = -3$

$\cot x = \frac{1}{2}$ means $\tan x = 2$: $x = 63.4°, 243.4°$

$\cot x = -3$ means $\tan x = -\frac{1}{3}$: $x = 161.6°, 341.6°$

Answer: x = 63.4°, 161.6°, 243.4°, 341.6°

8 Work-energy on a rough slope (CIE)

A 8 kg block is released from rest on a rough plane inclined at α, $\tan\alpha = \frac{3}{4}$, $\mu = 0.3$, and slides 5 m down a line of greatest slope. Use work and energy to find (a) the work done against friction, (b) its speed after 5 m. Take $g = 10$.

$R = 8g\cos\alpha = 64$ N, $F = \mu R = 19.2$ N
(a) Work done against friction $= F \times d = 19.2 \times 5$
(b) Loss of PE $= 8g \times 5\sin\alpha = 240$ J
Work-energy: $\frac{1}{2} \times 8 \times v^2 = 240 - 96 = 144$
$v^2 = 36$

Answer: (a) 96 J (b) 6 m s^{-1}

9 Particles on two smooth inclined planes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest. Find (a) the magnitude of the acceleration and which particle moves down, (b) the tension. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

A: weight component along plane $= 4g\sin 60°$; B: $8g\sin 45°$
B moves down, A up ($8\sin 45° > 4\sin 60°$): $F = ma$ for the system
$8g\sin 45° - 4g\sin 60° = (4 + 8)a$
(a) $a = 1.79$
(b) For A: $T - 4g\sin 60° = 4a$, so $T = 41.1$

Answer: (a) 1.79 m s^{-2}, B moves down (b) 41.1 N

10 Differential equations in context (All boards)

A drink at 92°C is left in a room at 22°C. Its temperature θ °C satisfies $\frac{d\theta}{dt} = -k(\theta - 22)$.

After 5 minutes the temperature is 67°C. Find k, then the time for it to reach 32°C, to 3 s.f.

$\int \frac{1}{\theta - 22}\, d\theta = \int -k\, dt$, so $\ln(\theta - 22) = -kt + C$

$t = 0$: $\theta - 22 = 70$, so $\theta - 22 = 70e^{-kt}$

$t = 5$: $45 = 70e^{-5k}$, so $k = \frac{\ln(\frac{14}{9})}{5} = 0.0884$

$10 = 70e^{-kt}$: $t = \frac{\ln(7)}{k} = 22.0$ minutes

Answer: k = 0.0884, t = 22.0 minutes