Day 165 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (2 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (3 kg), hanging 1.5 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8.

B: $3g - T = 3a$
A: $T = 2a$
Adding: $3g = 5a$, so $a = 5.88$
(a) $T = 2 \times 5.88 = 11.76$
(b) $v^2 = 0 + 2 \times 5.88 \times 1.5 = 17.64$
Answer: (a) 11.76 N (b) 4.2 m s^{-1}

2 Equations quadratic in disguise (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 5\sqrt{x} + 4 = 0$.

Let $u = \sqrt{x}$: $u^2 - 5u + 4 = 0$
$(u - 1)(u - 4) = 0$ so $u = 1$ or 4
$\sqrt{x} = 1$ gives $x = 1$; $\sqrt{x} = 4$ gives $x = 16$
Answer: x = 1 or x = 16

3 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix}2\\1\end{pmatrix}$ and $\begin{pmatrix}2\\-2\end{pmatrix}$. The point P lies on AB with AP : PB = 1 : 2.

Find the position vector of P and the exact distance OP.

$\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB} = \begin{pmatrix}2\\1\end{pmatrix} + \frac{1}{3}\begin{pmatrix}0\\-3\end{pmatrix}$

$= \begin{pmatrix}2\\0\end{pmatrix}$

$OP = \sqrt{2^2 + 0^2} = \sqrt{4} = 2$

Answer: $\begin{pmatrix}\mathbf{2}\\\mathbf{0}\end{pmatrix}$, OP = 2

4 Discriminant: range of k (All boards)

The equation $3x^2 + (2k - 2)x + (k - 1) = 0$ has real roots.
Find the set of possible values of k.

Discriminant: $b^2 - 4ac = (2k - 2)^2 - 4 \times 3(k - 1)$
$= 4k^2 - 20k + 16$
Roots of the quadratic in k: $k = 1$ and $k = 4$
Required: $b^2 - 4ac \geq 0$
Answer: $k \leq 1$ or $k \geq 4$

5 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 24 pairs of values of age of a car and its price, r = 0.582.
The critical value for a one-tailed test at the 1% level is 0.4716.
Test, at this level, whether there is positive correlation between the two variables.

$H_0: \rho = 0$, $H_1: \rho > 0$
Critical value 0.4716
$r = 0.582 > 0.4716$
This is in the critical region, so reject H_0.
There is evidence of positive correlation.
Answer: Reject H_0: evidence of correlation

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6 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 3315 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(150 + 105) × V
3315 = 127.5 V, so V = 26
(b) Deceleration = V ÷ 20 = 26 ÷ 20
Answer: (a) V = 26 (b) 1.3 m s^{-2}

7 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ -1 \end{pmatrix}$ and $\begin{pmatrix} k \\ 3 \\ 4 \end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 + 3k - 4 = 0$
$(k - 1)(k + 4) = 0$
$k = -4$ or $k = 1$. For $k = 1$ the first vector is $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, with length $\sqrt{3} = \sqrt{3}$
Answer: k = −4 or 1; length $\sqrt{3}$

8 Finding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(\mu, \sigma^2)$, with $P(X < 49) = 0.0668$ and $P(X > 61) = 0.3085$.
Find μ and σ.

z-values: 49: $z = -1.50$; 61: $z = 0.500$
$49 = \mu - 1.50\,\sigma$ and $61 = \mu + 0.500\,\sigma$
Subtract: $12 = 2.00\,\sigma$, so $\sigma = 6.00$
$\mu = 49 + 1.50 \times 6.00 = 58.0$
Answer: μ = 58, σ = 6

9 Tangent through the origin (All boards)

The tangent to the curve $y = e^x$ at the point P passes through the origin.
Find the coordinates of P and the equation of the tangent.

$\frac{dy}{dx} = e^x$; at $x = a$ the tangent is $y - e^a = e^a(x - a)$
Through (0, 0): $-e^a = -a\,e^a$, so $a = 1$ and $a = 1$
P = (1, e) and gradient $ke = e$
Answer: P(1, e); y = ex

10 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1 + x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.
(b) Use $x = 0.01$ to estimate $\sqrt{101}$ to 3 decimal places.

$(1 + y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, $y = 1x$

(a) $1 + \frac{1}{2}x - \frac{1}{8}x^2$

$x = \frac{1}{100}$: $101 = 10^2(1 + \frac{1}{100})$ so $\sqrt{101} = 10 \times (1 + \frac{1}{100})^{\frac{1}{2}}$

$\approx 10 \times (1 + \frac{1}{200} - \frac{1}{80000}) = 10.049875$

Answer: (a) $1 + \frac{1}{2}x - \frac{1}{8}x^2$ (b) 10.050