Day 164 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Inverse normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the height (cm), is N(181, 7^2). The highest 5% of values exceed k.
Find k to 3 significant figures.

P(X > k) = 0.05, so P(X < k) = 0.95
$z = \Phi^{-1}(0.95) = 1.64$
$k = 181 + 1.64 \times 7 = 193$
Answer: 193

2 Double angle equations (All boards)

Solve cos 2x + 2sin x = 1
for $-180° \leq x \leq 180°$.

$\cos 2x \equiv 1 - 2\sin^2 x$: $2\sin^2 x - 2\sin x = 0$
Solve the quadratic: sin x = 0 or sin x = 1
sin x = 0: x = −180°, 0°, 180°
sin x = 1: x = 90°
Answer: x = −180°, 0°, 90°, 180°

3 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 7 m s^{-1} and accelerates uniformly. She travels 127.2 m in the next 12 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $127.2 = 7\times12 + \frac{1}{2}\times a\times12^2$, so a = 0.6
(b) $v = u + at = 7 + 0.6\times12 = 14.2$
(c) For the midpoint, s = 63.6: $v^2 = u^2 + 2as = 7^2 + 2\times0.6\times63.6$
$v^2 = 125.32$
Answer: (a) 0.6 m s^{-2} (b) 14.2 m s^{-1} (c) 11.2 m s^{-1}

4 Triangle on a coordinate grid (All boards)

The diagram shows triangle ABC.
(a) Find the equation of the line through C perpendicular to AB.
(b) Hence find the coordinates of D, the foot of the perpendicular from C to AB, and the area of triangle ABC.

Gradient AB = 0, so gradient of the perpendicular = undefined (vertical)
Through C: x = 1
Line AB: y − 4 = 0(x − −1). Solve to get D = (1, 4)
Area = ½ × AB × CD = 12
Answer: (a) x = 1 (b) D(1, 4), area 12

5 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

f(x) = 4 arcsin(5x).

State the domain and range of f, and find $f^{-1}(x)$.

arcsin has domain −1 to 1, so $-1 \leq 5x \leq 1$

Range of arcsin is $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, so f(x) runs from $-\frac{4\pi}{2}$ to $\frac{4\pi}{2}$

y = 4 arcsin(5x) gives $\sin\frac{y}{4} = 5x$, so $f^{-1}(x) = \frac{1}{5}\sin\frac{x}{4}$

Answer: domain $-\frac{1}{5} \leq x \leq \frac{1}{5}$, range $-\frac{4\pi}{2} \leq f(x) \leq \frac{4\pi}{2}$, $f^{-1}(x) = \frac{1}{5}\sin\frac{x}{4}$

6 Discriminant: range of k (All boards)

The equation $3x^2 + (2k - 6)x + (2k - 6) = 0$ has real roots.
Find the set of possible values of k.

Discriminant: $b^2 - 4ac = (2k - 6)^2 - 4 \times 3(2k - 6)$
$= 4k^2 - 48k + 108$
Roots of the quadratic in k: k = 3 and k = 9
Required: $b^2 - 4ac \geq 0$
Answer: $k \leq 3$ or $k \geq 9$

7 Greatest and least values using R-form (All boards)

$g(x) = \frac{1}{30 + 21 \sin x + 20 \cos x}$
(a) Write 21 sin x + 20 cos x in the form R sin(x + α), with α acute.
(b) Find the greatest and least values of g(x) and the smallest positive x at which the greatest value occurs (1 d.p.).

$R = \sqrt{21^2 + 20^2} = 29$, $\alpha = \tan^{-1}\frac{20}{21} = 43.6°$
Denominator = 30 + 29 sin(x + 43.6°), ranging from 1 to 59
Greatest g = $\frac{1}{1}$ (denominator least), least g = $\frac{1}{59}$
Least denominator when sin(x + 43.6°) = −1: x + 43.6° = 270°
x = 226.4°
Answer: R = 29, α = 43.6°; greatest $\frac{1}{1}$, least $\frac{1}{59}$; x = 226.4°

8 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Parts are defective with probability 0.15, independently. A box of 10 parts is rejected if it has at least 3 defective.
From 6 such boxes, find the probability that exactly 3 are rejected (3 s.f.).

Y ~ B(10, 0.15). P(rejected) = P(Y ≥ 3) = 1 − 0.820 = 0.180
W ~ B(6, 0.180) is the number rejected
$P(W = 3) = C(6, 3)(0.180)^3(1 - 0.180)^3 = 0.0641$
Answer: 0.0641

9 Modulus inequalities (All boards)

Solve $|x - 8| < |2x - 8|$.

Square both sides: $(x - 8)^2 < (2x - 8)^2$
$((x - 8) - (2x - 8))((x - 8) + (2x - 8)) < 0$
$(-x)(3x - 16) < 0$
Critical values x = 0 and x = $\frac{16}{3}$; sketch the (positive or negative) quadratic.
Answer: $x < 0$ or $x > \frac{16}{3}$

10 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take g = 9.8. Give answers to 3 s.f. if not exact.

sin α = 0.6, cos α = 0.8; friction F = μR = 0.5×6 g cos α
B: 7 g − T = 7 a
A: T − 6 g sin α − F = 6 a
Adding: g(7 − 3.6 − 2.4) = 13 a
(a) a = 0.754; (b) T = 7(g − a)
Answer: (a) 0.754 m s^{-2} (b) 63.3 N